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Mukhopadhyay, Tridasnath, Ph.D.

The University of Michigan, 1987

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EVALUATION OF MANAGEMENT INFORMATION SYSTEMS:

A MICROECONOMIC APPROACH

by

Tridasnath Mukhopadhyay

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Business Administration)
in The University of Michigan
1988

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For My Parents

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CHAPTER 1
INTRODUCTION

The productivity of management information systems [MIS] has consistently figured among the top ten issues for information systems management [Ball 1982, Dickson 1984, Herbert 1986]. However, current approaches to MIS evaluation focus either on computer operations [Hamilton 1981], or on user satisfaction [Bailey 1983], and are not useful in estimating the effects of MIS on firm productivity [Stabell 1982]. To fill this gap, this research develops a microeconomic approach for MIS evaluation.

The proposed approach of MIS evaluation is based on microeconomic production theory. It views an MIS as part of a decision production process. According to this view, decisions are produced in much the same way as normal goods and services [Cooper 1983]. It is hypothesized that the production of decisions takes place in two stages. First an MIS converts raw data into information useful for decision making. Next information is entered into a decision model to produce decisions. At each stage of production, various types of labor and capital are employed to effect the transformation process.

This chapter introduces the research problem and summarizes the contribution and applicability of this research. First, a definition of MIS is given in Section 1.1. The need for MIS evaluation is examined in Section 1.2. Next, the contribution and applicability of this research

are discussed in Sections 1.3 and 1.4 respectively. Finally, an overview of this dissertation is given in Section 1.5.

1.1 Definition of an MIS

An MIS is an integrated user-machine system that provides information to support one or more decision making functions in an organization [Davis and Olson 1985, p.6]. It utilizes computer hardware and software, data and models, people (e.g., system analysts, programmers, computer operators, etc.) and manual procedures. Typical examples of MIS include sales forecasting and analysis, cash flow analysis, and production and inventory control systems.

An MIS should be distinguished from an organizational information system. Typically an organizational information system is a confederation of many interrelated management information systems [Senn 1978, Neumann 1980]. That is, an MIS is a part of an organizational information system, and may have varying degrees of linkages with other MIS in the organization.

1.2 Need for MIS Evaluation

The use of computer based systems for managerial decision making has increased significantly since computers were introduced into the business community during the 1950's. However, there is a definite lack of systematic procedures for MIS evaluation [Hamilton 1981]:

Evaluation of management information systems is an integral part of the management control process... Yet few organizations have an organized process for evaluating MIS effectiveness.

Although considerable work has been done on the provision of information efficiently, and on the impact of user needs, very little

attention has been paid to MIS evaluation [King and Rodriguez 1978]:

Both the literature of the MIS field and the day-to-day pronouncements of managers are replete with evaluations, many of which are negative, of information systems that have been developed to aid managers in performing their jobs. There is, however, a real dearth of scientific literature involving the systematic evaluation of information systems.

The current approaches of MIS evaluation focus either on computer operations [e.g., Hamilton and Chervany 1981], or on user satisfaction [e.g., Bailey 1983], and thus are not useful in estimating the effects of MIS on firm productivity [Stabell 1982]. In the absence of any approach to estimate the effects of an MIS on firm productivity, system use becomes a goal in itself. As a result, the productivity of MIS resources is often taken for granted [Kriebel et al. 1976, p.1]:

The world wide investment today in computing and information services to support organizational activities is measured in billions of dollars. Despite this fact, the productivity of these resources is an open question for all practical purposes.

The problem of MIS evaluation becomes more serious if one considers the trends in information technology. While the relative efficiency of information technology continues to increase, the real investment in this technology is also increasing with time. Despite the increasing reliance on information technology for managerial decision making, very little work has been done to estimate the impacts of MIS on firm productivity, and to aid management in making decisions about information technology [Chismar and Kriebel 1985].

1.3 Contribution of the Research

The review of the MIS literature above reveals a need to estimate the effects of MIS on firm productivity. In response to this need, a microeconomic approach is developed viewing decision making as a part of

the firm production system. The process of decision making itself is modeled as a two stage production process. First, an MIS converts raw data into information useful for decision making. Next, information is entered into a decision model to produce decisions. A metric for data, information, and decisions as inputs and outputs of this production model is developed. Certain criteria for the comparison of alternative MIS are also proposed.

The proposed approach has both descriptive and normative uses. As a descriptive tool, it enables managers to systematically generate alternative MIS designs, and examine the effects of each alternative on decisions as well as on firm output. As a normative tool, it allows managers to compare various MIS designs based on their effects on firm productivity.

The proposed approach has been operationalized and validated in the context of a fixed reorder cycle inventory control system. This part of the research also makes a contribution to the inventory control literature by examining the effects of information on inventory control decisions, and on firm output.

The neoclassical viewpoint used and validated in this research provides valuable guidelines for MIS evaluation. First, the process of building a model for MIS evaluation is facilitated by procedures for the identification and measurement of inputs and outputs. Next, a variety of production functions and their estimation procedures from the econometric literature can be used in the model estimation stage. Finally, the criteria of relative efficiency, effectiveness, and productivity proposed in this research can be used to establish a weak ordering of any set of MIS design alternatives.

1.4 Applicability of the Research

Since the proposed approach of evaluation focuses on the decision production process within a firm, it is not applicable to production and distribution of information to agencies external to an organization. For example, this approach is not appropriate for the evaluation of systems that produce information for customers, suppliers, investors, government agencies, etc.

A second limitation of this approach is due to microeconomic theory, which is the theoretical framework used. While neoclassical production theory leads to a theoretically sound approach for MIS evaluation, it also restricts the applicability of this work. The application of neoclassical theory requires that the output of the production process be sufficiently homogenous. This condition implies that the decisions made are relatively structured and involve operation and management control activities [Cooper 1983]. However, this is a minor limitation, because the majority of MIS are used for routine and repetitive decisions only [Gorry and Scott Morton 1971, Kleijnen 1980, p. 191].

1.5 Overview of the Dissertation

This dissertation is organized as follows. In Chapter 2, four streams of prior research and their implications on MIS evaluation are discussed. The microeconomic approach of MIS evaluation is presented in Chapter 3. Based on this approach, a neoclassical production model is developed for a fixed reorder cycle inventory system in Chapter 4. A simulation is designed in Chapter 5 to analyze this neoclassical model. Finally, the conclusions of this research are summarized in Chapter 6.

CHAPTER 2

PRIOR RESEARCH

Research relevant to the evaluation of information systems can be classified into four distinct categories:

- o Current Methods for MIS Evaluation
- o Information Economics
- o Measurement of Information
- o Decision Production Approach

The objective of this chapter is to critically review each of the four categories of research, and point out the ways in which the present research draws from and extends this literature. Finally, a summary of this review is given in the last section.

2.1 Current Methods for MIS Evaluation

An examination of current literature reveals six different methods for MIS evaluation. Each of these methods is reviewed below. A summary of these methods is given at the end of this section.

1. Computer System Approach: This method is concerned with the evaluation of computer systems. Several techniques are available for this purpose. For example, analytical modeling is often used for computer performance evaluation, and is well suited for design considerations such as the queuing analysis of an on-line system [Lucas

1986, Ch 13]. Special purpose simulation packages such as SCERT (Systems and Computers Evaluation and Review Technique) and CASE (Computer-Aided System Evaluation), on the other hand, enables modeling of systems that are too complex to allow precise mathematical formulation [Gotlieb 1985, Ch. 1]. However, the technique that has received consistent use as a computer performance evaluation tool involves the use of computer programs classified as benchmarks and synthetic modules [Lewis and Crews 1985]. Finally, hardware, software and hybrid monitors are also used to measure different performance indicators of a computer system.

The computer system approach has been used to some extent for MIS evaluation. This approach is implemented by defining certain criteria, and then comparing the performance of an MIS against a predetermined standard for each criterion. Hamilton and Chervany [1981] have surveyed some of the methods of this type. Quality assurance review, for example, focuses on the technical quality of an MIS. The production efficiency of a computer system is measured by percent uptime, actual throughput, and I/O channel utilization. Similarly, service level assessments are made using information on turnaround times, response times, and error rates.

The focus of this approach is to evaluate the computer operations of an MIS. Although this method is useful for a computer system, it is based on a narrow view of an MIS. It assumes that all is well with an MIS when the computer system works well. In other words, it does not consider the effects of information on decision making and firm productivity.

2. Cost-Benefit Analysis: Cost-benefit analysis requires the estimation of certain costs and gains that would result from alternative

courses of actions [McKean 1975]. The analysis of costs and benefits of MIS projects may present two different levels of difficulties depending on the types of benefits involved: Cost reduction and value enhancement [Emery 1974]. Cost reduction projects generate monetary benefits by lowering information processing costs. The benefits obtained from a cost reduction project may be in terms of savings in clerical labor, computer rental, etc. Since the costs and benefits of cost reduction projects are relatively easy to estimate, the analysis of such projects is quite straight forward. The objective of value enhancement projects, on the other hand, is not to reduce MIS costs, but to enhance the value of the system by generating benefits that result in the improvement of the operations of the organization. Value enhancement projects may lead to inventory reduction, higher capacity utilization, etc. However, the analysis of value enhancement projects may pose considerable difficulty if certain benefits can not be measured in monetary terms. Subjective judgments have to be made in order to deal with such benefits.

The common problems of Cost-benefit analysis "arise from incomplete identification of alternatives, cost accounting, assigning benefits, special characteristics of information systems, the cost of the analysis itself, and such realities as the local political and social environment" [King and Schrems 1978]. Moreover, Cost-benefit analysis does not examine the effects of information attributes such as accuracy, age, detail, etc. on firm productivity.

3. System Usage Approach: In the absence of a direct approach to estimate the effects of MIS on firm productivity, some researchers have taken system usage as an indicator of MIS success [e.g., King and

Rodriguez 1978, Lucas 1975, Fuerst 1979]. It is argued that system usage can be used as an indicator of MIS success under certain conditions such as voluntary usage. Ein-Dor and Segev [1978], for example, claim that "a manager will use a system intensively only if it meets some of the criteria (of success), and that use is highly correlated to them". Common measures of system usage include frequency of use, time per session, number of reports generated, and type of user (light, average, or heavy) [Srinivasan 1985].

The system usage approach has certain shortcomings. System usage may not be a suitable indicator of MIS success if the user has "motivations for using the system other than its objective utility in decision making (e.g., mandate from management, political motivation, self-protection for justifying 'poor' decisions)" [Ives et al. 1983]. Moreover, the link between system usage and the quality of decision making is a weak one [Ginzberg 1978]. If the system is considered as a service that enables managers to make decisions more effectively, the degree of system usage may not be the appropriate measure for MIS evaluation. In short, system usage is a necessary but not a sufficient condition for MIS success.

4. User Oriented Approach: The many MIS failures of 1960's [Ackoff 1967] gave rise to a serious concern for the user community. The user oriented approach views the purpose of an information system as providing computing services to a group of users, and thus it is based on user satisfaction. The proponents of user satisfaction argue that user satisfaction is correlated to information system utilization and system success [Bailey and Pearson 1983], and therefore can be used for MIS evaluation.

Several attempts have been made to operationalize the construct of user satisfaction. Gallagher [1974], for example, asked respondents to estimate the dollar value of a report, and rate the report on fifteen semantic differential scales. Neumann and Segev [1980], used four factors to measure user satisfaction: accuracy, content, frequency, and recency. Similarly, Bailey and Pearson [1983] have identified thirty-nine factors affecting user satisfaction, and rated them in an experimental study. Ives et al. [1983] have further tested this instrument, and have also developed a short-form of this instrument for research requiring only a global indicator of user satisfaction.

An important weakness of the user oriented approach is that the user satisfaction construct lacks a concise conceptual definition [Chismar et al. 1985]. As a result, several methods have been used in the literature to measure user satisfaction. The factors used by these methods are very diverse, and thus may not measure the same phenomenon. Moreover, many of these instruments are either unreliable, or lack careful validation [Ives et al. 1983].

The importance of user satisfaction for an MIS can hardly be overlooked. However, user satisfaction by itself is not a sufficient criterion for MIS productivity. For example, in an experimental gaming situation involving operations management decision making, Chervany and Dickson [1974] observed that subjects having raw data had higher decision confidence (satisfaction), but incurred higher total production costs than subjects receiving summary data. In this case, the information system providing raw data results in higher user satisfaction but lower productivity. In summary, there is no evidence of a causal relationship between user satisfaction and user performance,

and thus the user satisfaction construct should not be used as an outcome variable for an MIS [Chismar and Kriebel 1985].

5. Multi-Attribute Utility Approach: This method also employs the user's perspective in evaluating information systems. For example, Ahituv [1980] has proposed a multi-attribute utility function to assess the value of information systems. He considers three specific attributes - timeliness, content, and format - and makes certain assumptions regarding the relationship between utility and levels of each attribute. For an ideal situation, he considers an additive utility function for the three attributes mentioned above.

In the simple case of a single user, the multi-attribute utility approach provides a theoretical base for the evaluation of an MIS based on the user's perspective. However, this approach has certain practical problems. For example, the construction of a utility function requires a comprehensive list of attributes and valid measure for each attribute. Moreover, the use of a particular form of a utility function (e.g., an additive function) makes certain implicit assumptions regarding the underlying preferences of a user. If an MIS serves multiple users, this approach loses much of its relevance because it is difficult to assign a common set of preferences to a group of individuals [Arrow 1963].

6. Economic Production Analysis Approach: This method uses microeconomic theory to arrive at productivity measures of information systems. There are at least three examples of this approach in the literature. The first example is from Kriebel and Raviv [1980] who view the supply of computing and information services as a production

process. They take an "engineering approach" to relate the output of a computer system (standard job units of specific types per unit time) with different input resources through a production function characterized by fixed input coefficients. They define the productivity of a computer system in terms of various measures of production efficiency based on this model.

The second example is taken from Stabell [1982] who attempts to assess the effects of office technology on firm productivity. Stabell defines the firm as a set of information processing and realization activities. He then proposes the technical efficiency of realization activities as a measure of office productivity.

The third example in this category comes from Chismar and Kriebel [1985]. They use microeconomic production frontiers to compare output performance of business units based on the method of data envelopment analysis. With expenditures on information technology classified as separate input factors, they describe methods for analyzing business unit performance based on production efficiency.

Research on the productivity analysis of information systems has begun only recently. Although the examples cited above indicate considerable progress, research to date has not addressed the evaluation of MIS explicitly. For example, the productivity measures introduced by Kriebel and Raviv are useful for computer systems, but they have limited value for the purpose of MIS evaluation because they focus only on the computing aspects of an information system and not on the effects of information on decision making. The unit of analysis for the two other examples is at the firm level. Thus the last two examples are applicable to organizational information systems and not to specific MIS.

Summary: The current approaches to MIS evaluation have certain deficiencies. The system usage and the user oriented approaches are not theoretically well grounded. Ahituv's multi-attribute utility model and the computer based approach, on the other hand, are based on restrictive views of MIS. Similarly, cost-benefit analysis, although a useful tool for MIS project selection, does not attempt to examine the effects of information attributes on firm output. Finally, the economic production analysis approach holds much promise but has not yet explicitly addressed the evaluation of MIS.

The proposed method of MIS evaluation attempts to fill this gap in MIS research by examining the effects of information on decision making and firm productivity. This method is grounded in microeconomic production theory, and thus can be classified as an example of economic production analysis approach above. Since cost-benefit analysis is also based on microeconomics, this research shares the fundamental notion of comparing costs with benefits of alternative information systems with cost-benefit analysis. As stated earlier, this research is related to three other categories of research. The next stream of research to be reviewed is Information Economics.

2.2 Information Economics

Considerable work has been done to determine the value of information. The information economics literature, for example, provides a theoretical model for the determination of the value of information. A general consensus of this research is that the value of information is related to decisions [Davis and Olson 1985, p. 201]. Most research on information value is based on statistical decision theory. Therefore

this review begins with a summary of this theory.

Statistical decision theory [Raiffa 1970] provides guidelines for making decisions under uncertainty. The decision maker, who is an expected utility maximizer, has knowledge of payoffs for every action-state pair, but has only probabilistic knowledge of which state will occur. Information is used to refine the probabilistic knowledge of state occurrence in accordance with Bayes' theorem of conditional probability. Gains in expected utility due to revised knowledge of states defines information value.

Information Economics employs statistical decision theory to compare alternative information systems [Demski 1980]. Marschak and Radner [1972] extend the information economics approach to a team of persons. In their team theory, each agent decides his or her own acts, but everyone receives a common reward as the joint result of all their decisions. Feltham [1968], on the other hand, introduced a dynamic information economics model in which a memory component is incorporated to make learning and adaptive behavior possible.

Information Economics is based on certain restrictive assumptions [Treacy 1981]. It requires complete knowledge of possible states of nature and conditional probabilities of obtaining each signal given a state of nature. For the determination of information value, the information / action environment must be fully specified. Therefore, all relevant variables, relationships, and parameter values must be known a priori. Moreover, information economics does not explicitly concern itself with computer based management information systems [Keen 1982].

The operationalization of the information economics approach poses additional difficulties. First a great deal of input data is required

for this approach. Second, the computations can become tedious and involve difficult mathematical problems for certain statistical distributions. As a result, the information economics approach has found limited application for information system evaluation mainly for non-repetitive, one-shot decisions, occurring at the strategic level [Kleijnen 1980, p.191]. However, most MIS are built for repetitive use so that the information economics approach becomes too restrictive.

Summary: Although this dissertation does not employ information economics for the evaluation of an MIS, it uses some of the concepts of this approach. In particular, the focus on payoff as a means to evaluate information is a concept directly applicable to MIS. In general terms, payoff is measured by the increase in utility resulting from the outcome of an action (decision) and the revealed state of nature [Hilton 1981]. For MIS evaluation, it seems appropriate to define payoff in terms of factors relevant to the performance of a decision making function. This, however, requires the assumption that decisions are made for a specific purpose which has a predetermined measure [see Section 3.3].

2.3 Measurement of Information

A major problem with the evaluation of information systems is that there is no established method for measuring information [Mason 1978]. However, two approaches of information measurement can be identified in the literature. These two approaches are reviewed below:

1. Communication theory: According to communication theory [Shannon 1948], the amount of information is equal to the average number of

binary digits which must be transmitted to identify a given message from the set of all possible messages to which it belongs. Every time the number of potential messages is reduced by half, one unit of information is gained. This unit is called one "bit" of information.

A formula for measuring information in a data set is given by Theil [1969]. If there exists a set of n mutually exclusive hypotheses H_1, H_2, \dots, H_n , with prior probabilities p_1, p_2, \dots, p_n , where $\sum p_i = 1, 0 \leq p_i \leq 1$, then the average amount of information (I) contained in the data which transforms the prior probabilities p_1, p_2, \dots, p_n to the posterior probabilities q_1, q_2, \dots, q_n is given by

$$I = \sum q_i [\log_2 (q_i/p_i)] \text{ where } \sum q_i = 1, 0 \leq q_i \leq 1.$$

The unit bit developed in communication theory is useful in determining how efficiently the symbols to be communicated can be encoded. However, it does not explicitly attempt to analyze how precisely the transmitted symbols convey the desired meaning or how effectively the received meaning affect conduct in the desired way [Weaver 1949]. Moreover, the operationalization of this approach may pose considerable difficulty since it requires knowledge of all possible hypotheses and the estimation of prior and posterior probabilities for these hypotheses.

Shannon proposed the unit bit to address the technical problem of symbol transmission. A bit merely measures the amount of information in the sense of uncertainty reduction. So the amount measured does not specify the "content, value, truthfulness, exclusiveness, history or purpose of the information" [Miller 1953].

2. Multiple Attributes of Information: This method is based on the

assumption that a single measure of information is not adequate for most situations [Emery 1971]. A variety of attributes have been proposed in the literature. For example, Snavely [1967] suggested six attributes of information: relevance, reliability, understandability, significance, sufficiency, and practicality. Zmud [1978], on the other hand, derived four dimensions of information using factor analysis on a collection of adjectives rated by a group of subjects on a semantic differential scale. The derived dimensions include the following: an overall view of the quality of information consisting of a measure of relevancy, relevancy components, a measure of quality of presentation, and a view of the quality of meaning provided by information.

One weakness of the multiple attributes approach is that the attributes used are often not independent of each other. There is also a lack of objective measures for the attributes proposed in the literature. Moreover, there is no standard set of attributes available for use.

Summary: The measurement of information is a difficult task. A single measurement scheme may not be useful for all possible situations. Thus an alternative approach is to specifically design a measurement scheme for a given class of problems. For the purpose of MIS evaluation, therefore, this research develops a specific scheme for the measurement of various states of information in the decision production process.

2.4 Decision Production Approach

The decision production approach proposed by Cooper [1983] provides a framework for analyzing the effects of MIS on firm productivity.

According to this approach, decisions are produced in much the same way as normal goods and services. The physical resources used in the production process can be classified into labor (services of system analysts, programmers, staff analysts, managers, etc.) and capital (usage of computer hardware and software, communication devices, etc.). As with normal production, a transformation of raw material occurs in the decision production process. The raw material of decision production is information which is converted from its initial state (data) to its final condition (decisions).

Cooper views a decision production system as consisting of two components: a Mainline Component and a Management Control Component. The Management Control Component directs and coordinates the activities of the Mainline Component resources, and is peripheral to the process of decision making. The Mainline Component, on the other hand, is composed of four different processes. The first process is involved in the detection and selection of the External Data to produce Internal Data. The second process, Assimilation, interprets Internal Data, which results in an updating of Knowledge. Knowledge then becomes input to the Problem Structuring Process. The output, Problem Structure, is finally used by the Alternative Generation and Choice process to produce Decisions.

Cooper also proposes a measurement scheme for the different information states (External Data, Internal Data, Knowledge, Problem Structure, and Decisions) in the decision production process. This scheme characterizes information in terms of its age, detail, completeness, language, etc. The characterization scheme should be viewed as a guideline for the measurement of information states. The

actual scheme chosen should be dictated by the decision production system under study and the specific information state being examined.

The present research is based on the decision production approach. However, the decision production model used in this research differs from the four stage decision production model proposed by Cooper [1983]. This distinction can be attributed to the different objectives of the two research projects. The model proposed by Cooper attempts to explain management information requirements by studying the decision production process in detail. The primary objective of this research, on the other hand, is to examine the effects of MIS on firm productivity. Thus the scope of this research is not limited to the decision production process alone, it also extends to the effects of decisions on firm output. A modified model of decision production is, therefore, used to take into account the specific goal of this research.

Summary: The decision production approach introduced by Cooper is the foundation of this research. The goal of the original model proposed by Cooper is to study managerial information requirements. The objective of this research, however, is to evaluate MIS based on its effect on firm productivity. To accommodate this difference in objective, a modified model of decision production is used in this research.

2.5 Conclusion

A review of prior research relevant to this study is given in this chapter. An examination of the MIS literature shows that current approaches to MIS evaluation focus either on computer operations or on user satisfaction, and thus do not provide a comprehensive framework for

estimating the effects of MIS on firm productivity. The information economics literature, on the other hand, provides a useful theoretical model for one-shot strategic decisions. However, most MIS are used for repetitive decision making so that the information economics approach becomes too restrictive.

The decision production approach introduced by Cooper can be used as a framework for analyzing the effects of MIS on firm productivity. An advantage of this approach is that it enables the modeling of an MIS as part of the firm production process. It has also been shown in this chapter that there is no standard method for measuring information. A reasonable approach, therefore, is to develop a specific measurement scheme for a given class of problems.

The next chapter outlines the proposed approach of MIS evaluation. This approach defines two orthogonal dimensions for the measurement of different information states in the decision production process. It also presents certain criteria for the comparison of alternative MIS.

CHAPTER 3

A TWO STAGE MODEL OF DECISION PRODUCTION

A review of prior research on information system evaluation, presented in the previous chapter indicated that current approaches to MIS evaluation typically focus either on computer operations or on user satisfaction, and thus are not useful in estimating the effects of an MIS on firm productivity. To fill this gap, a neoclassical approach for MIS evaluation is developed in this research. In this chapter, this model of MIS evaluation is described, and certain criteria for the comparison of alternative MIS are discussed.

The proposed approach is based on the concept of a decision unit (DU) that produces one type of decision repeatedly. It is hypothesized that a DU makes decisions based on an underlying model of reality. It is postulated that the production of decisions in a DU takes place in two stages. First an MIS converts raw data into information useful for decision making. Next, information is entered into a decision model to produce decisions. At each stage of production, various types of labor and capital are employed to effect the transformation process.

The proposed approach to MIS evaluation, based on the two stage model of decision production, is presented in this chapter. This chapter is organized as follows. In Section 3.1, a measurement scheme for data, information, and decisions is described. The two stage decision production model and the assumptions used in this model are presented in

Section 3.2. Certain postulates are made in Section 3.3 to help model the behavior of a decision unit. To operationalize MIS evaluation, several criteria are discussed in Section 3.4 for the comparison of alternative MIS. Next, certain procedures for the identification and measurement of inputs and outputs of the decision production model are presented in Section 3.5. Finally, a summary of this chapter is given in Section 3.6.

3.1 Measurement of Data, Information, and Decisions

Data, information, and decision can be defined as inputs and outputs of the two stage decision production process:

1. *Data* is the raw material of decision production, and refers to the direct recording of entities or events occurring in reality. Data may be generated by events such as customer orders, customer payments, product shipments, and product receipts. Data may also be generated to describe entities such as customers, employees, inventory items, etc. In short, data constitutes an input to an MIS that produces information useful for decision making.

2. *Information* refers to the output of an MIS. The contents of an inventory master file, for example, become information when they are retrieved from a data base and presented as a report to a decision maker. The nature of information presented to a decision maker depends on the specific design of an MIS. An MIS may present detailed or summarized information about entities and events relevant to a decision problem, suggest alternative courses of action, or recommend a specific action. The decision maker, however, makes the final choice of actions.

3. *Decision* refers to the action alternative chosen by a DU. That

is, a decision is defined as the description of an action and not the action itself. For example, in an inventory control system, decision refers to the quantity ordered and not the implementation of the order.

Data, information, and decisions are considered as three different states of the same basic material of decision production. Thus a single scheme is deemed adequate for the measurement of all three states. The proposed scheme involves two orthogonal dimensions: accuracy and coverage. Since data, information, and decisions are essentially descriptions of objects and events in reality, the measurement of both accuracy and coverage entails an examination of the description contained in data, information, or decisions, as the case may be.

Accuracy refers to the extent to which a description is in accord with reality, whereas coverage is a measure of how inclusively it represents relevant parts of reality. A description contained in data, information, or decisions may not include all relevant parts of reality, and thus may not lead to full coverage. Given that a description represents some specific parts of reality (a certain level of coverage), the level of accuracy is assessed by comparing the image presented by the description with the corresponding part of reality.

To illustrate, consider the stock on hand information of items stored in a warehouse. Assume that the stock position of only half the items are available (although information on all items are required for decision making) thus leading to a coverage level of half. Given this level of coverage, the accuracy of the stock on hand information can be determined by comparing the reported stock with the actual stock held by the warehouse. The exact procedure for the calculation of accuracy will be described later in this section.

Often a description may represent an abstract concept, and thus can not be checked against reality. In that case, the description is compared with the description that a rational decision maker considers accurate when his knowledge of reality is accurate. For example, the accuracy of a decision is checked by comparing the selected decision alternative with the decision alternative that would be chosen by a rational manager when his decision model exactly replicates reality.

Note that coverage and accuracy are measured as average values across the decision period. Thus measurement of coverage and accuracy must take into account all decisions made during the production period. In the remainder of this section procedures for measuring coverage and accuracy are described in detail.

Measurement of Coverage: The measurement of coverage involves the measurement of five attributes of coverage. Two of these attributes relate to events occurring (or objects existing) in space, and are called spatial attributes. Three other attributes refer to the temporal aspects of reality, and are termed temporal attributes. Each of these attributes is defined below:

1. *Spatial detail* is the lack of aggregation in a description across events or entities. Spatial detail decreases if a description provides a summarized account of reality. For example, aggregate sales data exhibits a lower level of spatial detail than item level sales data.

2. *Spatial scope* is the number of events or entities included in a description of reality. Spatial scope diminishes if all relevant entities or events are not included in a description. For example, spatial scope of customer information decreases if information on some customers is not available.

The independence between spatial detail and spatial scope can be illustrated through a simple example. Consider, for example, stock on hand information generated by an MIS that describes a certain percentage of all inventory items in a warehouse. Given this specific level of spatial scope, stock on hand information may describe inventory items at different levels of aggregation thus leading to different values for spatial detail.

3. *Temporal detail* is the lack of aggregation in a description against time. Temporal detail is determined by the frequency with which a description tracks reality. For example, daily sales data exhibits higher level of temporal detail than monthly sales data.

4. *Temporal scope* refers to the length of time for which a description tracks reality. Temporal scope of a description increases if it tracks reality for a longer time span. For example, a file containing transaction data for one year has a higher temporal scope than a second file containing data for a month.

The independence between temporal scope and temporal detail can also be illustrated through an example. Consider a situation where sales data are available for a year. Given this level of temporal scope, the sales data may be represented at different levels of aggregation such as daily, weekly, etc. leading to different levels of temporal detail.

5. *Timeliness* refers to the age of a description. The age of a description can be reduced by changing the design of an MIS. For example, changing a batch system to an on-line system can improve the timeliness of a description. However, the exact age of a description can only be determined by analyzing detailed information flow in a system. Miller and Strong [1986], for example, discuss how network models of

information flow can be used to determine the age of information.

Timeliness can also be illustrated as an independent attribute. Consider a management report that contains daily sales data (a specific level of temporal detail) for five days (a specific level of temporal scope). The age of this report may, however, vary depending on the responsiveness of the MIS. For example, the report may contain one day old or five days old data leading to two different levels of timeliness.

Each of the five attributes of coverage is measured in relative terms. For example, the temporal detail of daily sales data is considered five times as much as weekly sales data assuming five working days in a week. The measurement of each attribute is normalized by dividing the raw score of an attribute by the maximum score possible for that attribute.

It may not always be necessary to consider all five attributes of coverage. One or more attributes of coverage may either be irrelevant or may have fixed values, and thus may be excluded from further consideration. For example, the number of items in an inventory control system may not change over the decision period leading to a constant level of spatial scope for decision coverage. Hence spatial scope may be excluded from the measurement of decision coverage.

Measurement of Accuracy: The accuracy of a description contained in data, information, or decisions is measured in relation to the absolute magnitude of the description. That is, if two descriptions have an identical error term, the description with lower magnitude is considered more accurate. Specifically, the accuracy of a description is measured according to the following scheme:

$$(3.1) \text{ Accuracy} = 1 - \text{Error} / \text{Absolute Magnitude};$$

where absolute magnitude of the description is estimated by the absolute value of the mean of the accurate quantities represented by the description during the decision production period. For example, if the accurate quantity on hand of an item is given by q_1, q_2, \dots, q_n during the n days in the production period, the absolute magnitude of the quantity on hand information can be estimated by $1/n[q_1 + q_2 + \dots + q_n]$ where $q_i \geq 0$.

The error term of a description may have three components:

1. *Random*: Random errors in a description may be caused by a variety of reasons. For example, random errors may occur at the time of recording data from one medium to another. Random errors can be traced to typical data processing errors such as transposition error, transcription error, logical error, measurement error, and sampling error. Random errors can also be caused due to incomplete knowledge of a stochastic process represented in a description of reality. For example, the estimation of product demand may include a random error component because of the lack of understanding of the demand distribution.

2. *Systematic*: Systematic errors in a description signal the presence of bias. For example, an incorrect assumption in a decision model may introduce bias in the decisions made. Systematic errors can also occur due to behavioral reasons. Sometimes subordinates may willfully present biased information to a manager in order to gain favor or protect themselves.

3. *Conflict*: Conflict refers to the contradictory evidence found in multiple descriptions of reality. An MIS exhibits no conflict if it contains a single description of reality. One effective method of lowering conflict is to use a data base management system to eliminate

redundant data.

The measurement of the error components can be facilitated by using standard statistical techniques. For example, systematic and random errors can be measured by the mean and standard deviation of the deviations of a description from reality. Similarly, if an information system contains two descriptions of reality, the covariance of the deviations of the two descriptions can be used as a measure of conflict. Higher the covariance, lower is the conflict between the two descriptions of reality.

Often the error term may not contain all three components described above. The measurement of the error term when one or more components have a zero value is illustrated in Table 3.1 below.

The notation for Table 3.1 can be explained as follows. Consider an MIS that contains two descriptions of some specific parts of reality. Let d denote the deviations (d_1, d_2, \dots, d_n) of one description from reality and d' the deviations $(d'_1, d'_2, \dots, d'_n)$ of the second description. For example, if the on hand quantities of an inventory item for n days are given by an inventory master file as q'_1, q'_2, \dots, q'_n and the actual on hand quantities are q_1, q_2, \dots, q_n , then the deviations of the quantity on hand information can be calculated as $d_i = q_i - q'_i$; $i = 1, 2, \dots, n$. The deviations of an alternative description of the quantity on hand information can be calculated in a similar way.

The measurement of the error term is illustrated in Table 3.1 for two cases: (1) an MIS with a single description of reality, (2) an MIS with two descriptions of reality. With a single description of reality, the error term becomes zero if the description does not deviate from reality ($d_i = 0$; $i = 1, 2, \dots, n$). Most often, however, the error term

would have a random, systematic, or both components. The conflict component, absent in the first case, would be present in the second case if the two descriptions do not match each other. The scheme given in Table 3.1 can be easily extended to situations where more than two descriptions are contained in an MIS.

TABLE 3.1: MEASUREMENT OF THE ERROR TERM

Number of Descriptions	Components of Error	Squared Error Term
One	Random	Variance (d)
One	Systematic	Squared Bias (d)
One	Random, Systematic	Mean Squared Error (d)
Two	Random, Conflict	$\text{Var}(d) + \text{Var}(d') - 2 \text{Cov}(d, d') $
Two	Systematic, Conflict	$[\text{Bias}(d)]^2 + [\text{Bias}(d')]^2 - 2 \text{Cov}(d, d') $
Two	All Three	$\text{MSE}(d) + \text{MSE}(d') - 2 \text{Cov}(d, d') $

A wide variety of methods can be used by management to improve the accuracy of data, information, and decisions. These methods can be classified into two categories. The first category of methods involve mainly data processing operations as they are used to improve the accuracy of data entry and file updating functions. Examples of data processing controls include batch counts, batch totals (for batch oriented systems only), check digits, range tests, existence tests, completeness tests, etc. [Senn 1984, Power et al. 1984]. Other data processing methods include source data capture with Optical Character Recognition, data entry through intelligent terminals, data base management techniques, etc. See Ballou and Pazer [1985] and Morey [1982] for some of the data quality control problems in multi-user

information systems.

The second category of methods concern logical analysis and modeling aspects of decision making. Selection of appropriate statistical, operations research, and other analytical methods may significantly impact the accuracy of decisions made. Strict quality assurance of end user developed applications may also improve information accuracy [Davis 1981]. Moreover, the skill and experience of the decision maker would affect the level of decision accuracy.

Finally it should be noted that the measurement scheme described here has certain limitations. One limitation of this scheme is that it is applicable to quantitative data only. Moreover, no claim is made about the completeness of this scheme. In particular, it does not include any behavioral attributes often used in the MIS literature [e.g., Bailey 1983], because this research is not an attempt at behavioral modeling.

The focus of the proposed scheme on quantitative data is not a serious limitation for MIS evaluation because quantitative data constitute the majority of all data contained in data processing systems. In addition, as pointed out earlier (Section 2.3), a single measurement scheme is unlikely to be useful for all possible problems. Given this difficulty, the proposed scheme is expected to be a useful tool for MIS evaluation, mainly due to its reliance on independent attributes and objective measures for each attribute.

The measurement of data, information, and decisions constitutes the first step toward MIS evaluation. The comparison of alternative MIS, however, requires an understanding of the two stage model of decision production. This model is discussed in the next section.

3.2 Two Stages of Decision Production

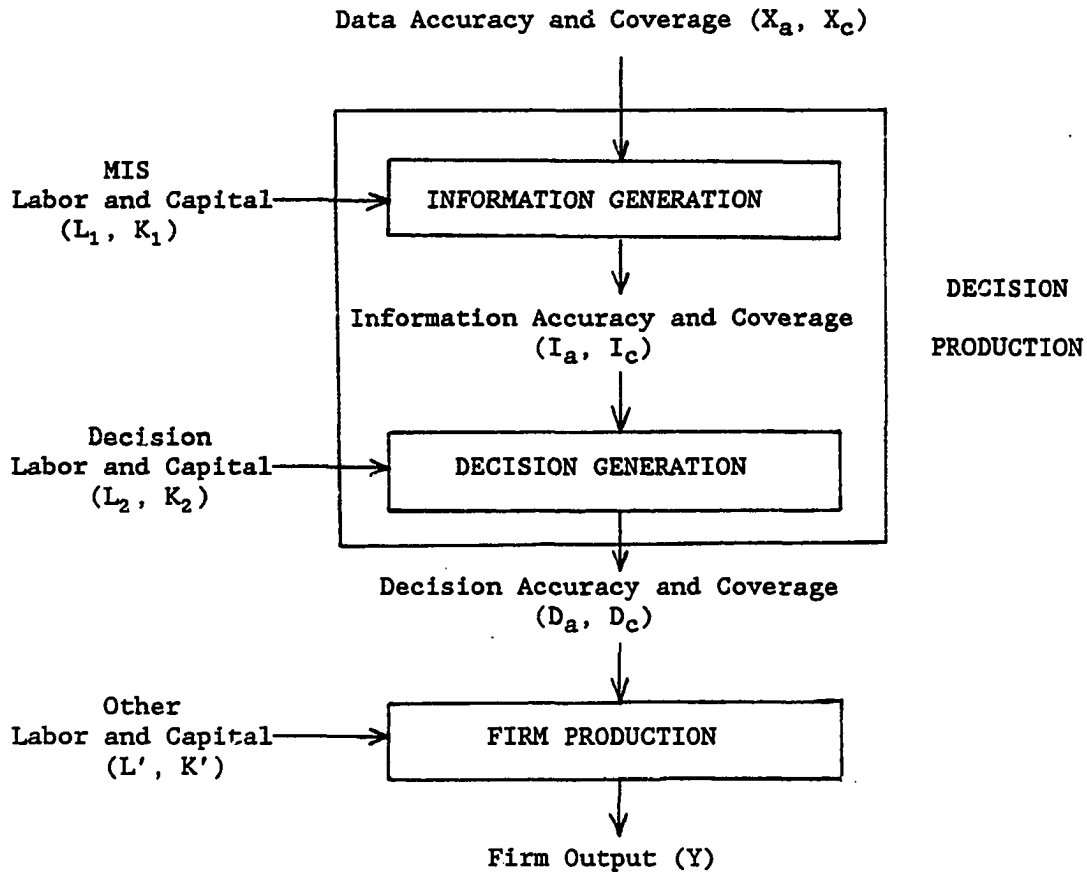
An important reason for the paucity of research on assessing the economic impact of MIS can be traced to a lack of understanding of the role of information systems in the firm production process. The decision production approach introduced by Cooper [1983], however, enables the development of a theoretical model linking information systems with the firm production process. Such a theoretical decision production model, developed in this research, is discussed in this section.

The production of decisions in a DU is viewed as a two stage process [Figure 3.1]. The first stage of this process entails the conversion of raw data into information, and is called Information Generation. In the second stage of decision production, Decision Generation, a decision maker uses the information generated in the previous stage to make decisions. Finally, the decisions produced by the DU are input into the firm's production system along with other labor and capital. The different stages of this production process are discussed below.

The description of the different stages of decision production given below is based on neoclassical production theory. Neoclassical theory mandates certain characteristics on the production process. There is indeed some evidence in the literature to support the efficacy of certain neoclassical assumptions for the decision production process (see Cooper [1983a] for a comprehensive discussion on this issue). While the discussion below uses the neoclassical framework, one purpose of this research is to empirically test some of the neoclassical assumptions that relate to the functional relationship between inputs and outputs. To clarify the stand taken by this research with regard to the neoclassical view, common neoclassical assumptions are listed at the

end of this section, and the applicability of each assumption for this research is also stated explicitly.

FIGURE 3.1: DECISION PRODUCTION



Information Generation: The process of transformation of raw data into information useful for decision making is called Information Generation. An MIS effects this conversion process using a variety of capital and labor inputs. The different types of capital inputs employed by an MIS may include computer hardware and software and communication networks. An MIS also typically uses the services of computer operators, data entry clerks, system analysts and programmers, and staff analysts in the Information Generation process.

The raw material of Information Generation is data. The accuracy and

coverage of data are determined by the way data is captured and entered into the MIS. Many design alternatives are available to management to control data accuracy and coverage. However, these design alternatives can be classified into two major groups. The first category of methods uses some source documents (e.g., standard forms to record customer orders, shipments, etc.) to capture data. Later one of many data entry devices (e.g., keypunch, key to tape, key to disk, optical character recognition, etc.) is used to transfer data into a computer readable medium. The second category of methods eliminates the source document by entering data into auxiliary memory as transactions take place (e.g., point of sale terminals, airline reservation, etc.). The exact levels of data accuracy and coverage are, however, determined by detailed design specifications of an MIS.

The Information Generation process for a general case can be described using a joint production function:

$$(3.2) F_1 (I_a, I_c, X_a, X_c, L_1, K_1) = 0;$$

where I_a and I_c denote information accuracy and coverage, X_a and X_c data accuracy and coverage, and the two vectors L_1 and K_1 represent various types of labor and capital employed by an MIS. Alternatively, the production function (3.2) can be written in the less symmetrical form:

$$(3.2a) I_a = f_1 (I_c, X_a, X_c, L_1, K_1).$$

The function F_1 , or f_1 , is defined as giving the maximum amount of any product - say, I_a - that can be produced from any feasible combination of I_c , X_a , X_c , L_1 , and K_1 .

It is expected that the marginal product¹ of each input is positive and diminishing in the relevant range of production. For example,

¹The marginal product of an input is the first order partial derivative of the production function with respect to the input.

increasing data accuracy, keeping all other inputs constant, is expected to increase information accuracy (for a given level of information coverage), although at a decreasing rate. However, equation (3.2) does not rule out some inefficient points from the production function: there may exist feasible points where some marginal products are negative.

It is expected that the outputs, as well as the inputs, are continuously substitutable. In other words, there is a negative tradeoff between each pair of outputs, as well as between each pair of inputs. For example, for a given set of inputs, the outputs I_a and I_c are expected to be continuously substitutable for each other.

Decision Generation: In the second stage of decision production, Decision Generation, information produced by the MIS is reported to a decision maker. The decision maker makes decisions based on an underlying model of reality. The division of tasks between the two stages of decision production depends on the design of an MIS. At the minimum level of support, an MIS may only store potentially useful information leaving the decision maker to determine the implications of the stored information on his decision problem [Mason 1981]. However, often an MIS may also analyze the stored data using an abstract model of reality, and make prediction about future states of nature, and / or suggest alternative courses of action. But the decision maker takes the final choice of action, and retains the power to reject any suggestions made by an MIS.

It may be noted that the potential level of support provided by an MIS is often determined by the decision problem on hand [Emery 1971]. For unstructured decision problems, for example, an MIS may only provide the minimum level of support (i.e., store potentially useful

information). For structured decisions, on the other hand, an MIS may be able to recommend a specific course of action.

The decision generation process for a general case can be described by a joint production function:

$$(3.3) F_2 (D_a, D_c, I_a, I_c, L_2, K_2) = 0;$$

where D_a and D_c refer to decision accuracy and coverage, and L_2 and K_2 represent decision labor and capital. The production function (3.3) can also be given in the less symmetric form:

$$(3.3a) D_a = f_2 (D_c, I_a, I_c, L_2, K_2).$$

The properties of production function (3.3), or (3.3a), are essentially the same as those of (3.2). For example, it is expected that the marginal product of each input of (3.3) is positive but diminishing in the relevant range of production. Similarly, any pair of inputs (outputs) is expected to be continuously substitutable for each other, *ceteris paribus*.

The two stages of decision production can also be combined using a single production function:

$$(3.4) F (D_a, D_c, X_a, X_c, L_1, L_2, K_1, K_2) = 0.$$

The production function given in (3.4) can be used to summarize the overall decision production process.

Firm Production: The decisions produced by the DU are finally input into the firm production system along with other labor and capital. For example, inventory control decisions are considered as one of many inputs into the production process of a manufacturing firm. In other words, the firm production process can be given as:

$$(3.5) Y = f (D_a, D_c, L', K');$$

where Y denotes firm output and L' and K' other labor and capital. The

properties of the firm production function (3.5) are expected to be similar to those of the Information Generation and Decision Generation functions described earlier.

The overall firm production process including the two stages of decision production can be summarized using a single production function:

$$(3.6) \quad Y = f' (X_a, X_c, L, K);$$

where the firm labor (L) and capital (K) are vectors $L [L_1 \ L_2 \ L']$, and $K [K_1 \ K_2 \ K']$. The decision production model described above uses certain neoclassical assumptions. In the remainder of this section, common neoclassical assumptions are listed, and their applicability in this research is examined.

Neoclassical Assumptions: The neoclassical production theory provides a useful framework for the decision production approach. However, the neoclassical view also imposes certain restrictions on this research. First, it requires that the output of the production process be sufficiently homogeneous (see also Section 1.4). As a result the proposed approach can be used to evaluate MIS involving routine and repetitive decisions (see Cooper [1983a] for a rigorous discussion of the applicability of the decision production approach). Second, the neoclassical framework makes certain implicit assumptions regarding the production process. The following discussion explicates the production function concept and the associated assumptions used in this research.

The decision and firm production functions given earlier [equations (3.2) through (3.6)] describe one common form. However, the concept of production function used in this research is more general. A specific production function for this model may be given by a single function, or

a system of equations. This definition, although perfectly general, is assumed to satisfy the following neoclassical assumptions:

1. The input requirement set is nonempty [Varian 1978, p. 6]. That is something can not be produced from nothing. For example, Information Generation is not feasible without data and MIS resources.

2. The production possibility set is bounded [Walters 1963]. In other words, there are limits to the inputs available and to the quantity of output produced. This assumption is based on the fact that the inputs and outputs are economic (scarce) resources, and can not be available in unlimited quantities. This assumption also applies to the information states involved in the decision production process. For example, the maximum level of an information state accuracy is limited to unity.

3. A production function indicates the maximum output possible from every input combination [Ferguson 1969, p. 7]. This assumption assures a single valued production function. For example, the levels of information accuracy and coverage are maximum possible for given levels of data accuracy and coverage, and MIS labor and capital.

4. Technology is assumed to be constant during the production period [Dano 1966, p.10]. That is, the set of inputs and outputs and the shape of the production remain unchanged during the production period. One implication of this assumption is that the MIS design does not change during the period under study.

5. Production is assumed to be continuous and repetitive during a production period, with consumption of nondurable inputs and services of durable inputs as well as output production occurring at constant rates [Dano 1966, p.8]. This assumption implies that the production process is

time invariant.

6. The production function is defined only for nonnegative values of inputs and outputs [Dano 1966, p.12]. This assumption indicates that negative values of inputs and outputs are meaningless. For example, negative values of accuracy and coverage for an information state are not allowed.

Two more assumptions, listed below, deserve special attention for MIS evaluation, because they relate to the functional relationship between inputs and outputs:

1. Marginal product of each input is positive and diminishing in the relevant range of production [Ferguson 1969, p.69].

2. Inputs are continuously substitutable in producing the same output level, and this substitution occurs at a diminishing marginal rate [Mansfield 1982, p.158].

The decision production model described in this section provides a foundation for MIS evaluation. However, certain postulates about the behavior of a decision unit are required to operationalize the proposed approach of MIS evaluation. Two assumptions about the behavior of a DU are discussed in the following section.

3.3 Behavior of a Decision Unit

Two postulates are required about the behavior of a decision unit to enable the evaluation of MIS. The first postulate proposes that a DU has a specific purpose, thus allowing the evaluation of a DU in the light of its purpose. The second postulate states that a DU maximizes its performance measure, thus enabling the comparison of alternative MIS in terms of this measure. Both these postulates are explained below.

Postulate 1: A DU has a specific purpose.

Postulate 1 rules out random behavior on the part of a DU. It does not allow a DU to produce arbitrary decisions. Postulate 1 also enables one to attach a value to the output of a DU. Since a DU has a specific purpose, there exists a measure of its performance [Ackoff 1971]. In the inventory control example, the purpose of a DU is to maximize its net contribution, where net contribution is total contribution minus inventory related costs.

It is not necessary that the purpose of a DU be expressed in terms of dollars. It is also feasible that a DU has a vector of sub-purposes. In this case, it will be assumed that the overall purpose of a DU is a function of its sub-purposes. For the sake of simplicity, however, this discussion will be restricted to DU's that have a purpose expressible in monetary units.

Given that a DU has a specific purpose, each decision made by a DU can be said to contribute to the attainment of the purpose of the DU. In other words, each decision is associated with a payoff which has the same cardinal measure as the purpose of the DU. In the inventory control example, the purpose of the DU is to maximize net contribution. In this case payoff of a decision can be expressed in terms of net contribution.

The gross payoff of a DU is the total payoff per unit time. For the inventory control example, gross payoff is the total net contribution for the production period. The cost associated with a DU, on the other hand, refers to the total cost of Information Generation and Decision Generation. The cost of decision production (C) can be given by the following relation:

$$(3.7) C = W_1 L_1 + W_2 L_2 + R_1 K_1 + R_2 K_2;$$

where W_1, W_2 refer to labor wages, R_1, R_2 represent rents for capital inputs. The net payoff for a DU is given by the gross payoff minus the cost of decision production.

Postulate 2: A DU maximizes its net payoff.

Postulate 2 enables the comparison of alternative MIS. For example, from a given set of alternative MIS, the MIS with the highest net payoff should be ranked first, followed by the MIS with next highest net payoff, and so on.

Postulate 2 implies that a DU uses that level of factor inputs which maximizes its net payoff. Using mathematical notation, the maximization problem of a DU can be given as:

$$(3.8) \text{ Maximize: } N - P - C$$

$$\text{Subject to: } F(D_a, D_c, X_a, X_c, L_1, L_2, K_1, K_2) = 0;$$

where N and P denotes net payoff and gross payoff per unit time.

Two weaker, though consistent, versions of Postulate 2 are possible. Each of these postulates is explained below.

Postulate 2A: A DU minimizes its cost given a level of gross payoff.

This postulate is comparable to a neoclassical cost minimization problem. In terms of mathematical notation, this problem can be stated as follows:

$$(3.9) \text{ Minimize: } C$$

$$\text{Subject to: } P = P_0,$$

$$\text{and } F(D_a, D_c, X_a, X_c, L_1, L_2, K_1, K_2) = 0.$$

Postulate 2B: A DU maximizes its gross payoff given a level of cost.

This postulate is equivalent to a neoclassical revenue maximization problem. Mathematically, this postulate can be expressed as:

(3.10) Maximize: P

Subject to: $C = C_0$,

and $F(D_A, D_C, X_A, X_C, L_1, L_2, K_1, K_2) = 0$.

The behavioral assumptions made about a DU in this section allows one to compare alternative MIS. Several criteria for the comparison of alternative MIS, based on the postulates described above, are developed in this research. Some of these criteria are discussed in Section 3.4.

3.4 Comparison of Alternative MIS

It is proposed that an MIS should be evaluated by comparing it with one or more alternative MIS. Several criteria are presented in this section for the comparison of alternative MIS. These criteria can be classified into two categories. While the first category is applicable to MIS that produce the same levels of information accuracy and coverage from different levels of MIS resources, the second set is used to compare MIS that produce different levels of information and coverage. Both types of criteria are explained below.

Let the Information Generation process in an existing MIS be given by the following function:

$$(3.11) F_1(I_A, I_C, X_A, X_C, L_1, K_1) = 0;$$

where I_A and I_C refer to the current levels of information accuracy and coverage generated from the given quantities of inputs: (X_A, X_C, L_1, K_1) .

Consider replacing the existing MIS with a new system. The following two types of impacts are expected in the decision production process due to a change in the information system:²

²Emery [1974] identified the two types of information system projects: Cost Reduction and Value Enhancement.

1. Cost Reduction occurs when certain aspects of an MIS are altered but the information generated remains the same. In terms of the two stage model, cost reduction implies a change in the productive efficiency of Information Generation. In other words, the new Information Generation process can be described as:

$$(3.12) F'_1(I_a, I_c, X_a, X_c, L'_1, K'_1) = 0.$$

Thus the new MIS produces the same levels of information accuracy and coverage (I_a and I_c) from different quantities and kinds of labor and capital; data accuracy and coverage are assumed constant. Examples of cost reduction include installation of a current generation machine, design of a more efficient sorting routine, automation of a manual process, alteration of the physical design of a database, etc.

The comparison of alternative MIS in case of cost reduction is relatively straight forward. Since cost reduction does not change the levels of information accuracy and coverage, it does not affect the Decision Generation process, and thus the comparison of MIS is limited to Information Generation. Based upon the microeconomic literature [Farrel 1957, Forsund et al 1980], three measures of relative efficiency are proposed to compare two MIS's, A and B, in case of cost reduction:

Relative Technical Efficiency of Information Generation: Let A and B produce the same amounts of information accuracy and coverage from different levels of input resources: $Z_A = [L_{1A}, K_{1A}]$ and $Z_B = [L_{1B}, K_{1B}]$. Then A is considered technically more efficient if and only if $Z_A < Z_B$. That is, A produces same information as B, but uses less of at least one input. In other words, A produces same outputs with less inputs. Consider, for example, the design of a more efficient file

organization for an MIS. The implementation of the new file organization leads to savings in auxiliary memory space (disk, tape, etc.) and computer connect time. Thus the new file organization decreases the level of capital input used resulting in higher technical efficiency.

Relative Economic Efficiency of Information Generation: Let A and B produce the same levels of information accuracy and coverage from different quantities of input resources Z_A and Z_B . Then A is economically more efficient than B if and only if $WZ_A < WZ_B$, where W is a vector of factor prices. Note that technical efficiency implies economic efficiency but not vice versa. This is because economic efficiency may also be due to allocative efficiency.

Relative Allocative Efficiency of Information Generation: Let A be economically more efficient, but not technically more efficient than B. Then A is considered allocatively more efficient than B because A employs an input proportion that leads to lower cost of Information Generation. The concept of allocative efficiency emphasizes the replacement of an input by a cheaper substitute. The rationale behind the automation of manual processes, for example, often involves the substitution of costly labor by cheaper capital equipment.

In summary, the criterion of relative economic efficiency can be considered as the most general rule for the comparison of alternative MIS in case of cost reduction since an improvement in technical or allocative efficiency always results in an increase in economic efficiency. An increase in economic efficiency, on the other hand, may be due to a gain in either technical or allocative efficiency, or both. The use of the two more specific rules (when applicable), however, allows one to identify the exact reason for efficiency gain.

2. Value Enhancement causes a change in the information generated by an MIS. In terms of the decision production model, value enhancement refers to a change in the output of the Information Generation process. The change in information accuracy and / or coverage may be due to a change in the levels of input resources L_1 and K_1 and / or a change in the functional form of (3.11). For example, additional input resources may be used to increase the spatial detail of information leading to a higher level of information coverage. Or, a centralized data base may be designed to reduce conflict between different descriptions of information, and thus improve information accuracy. However, the implementation of such a data base alters the technology of Information Generation resulting in a different functional form for (3.11).

The impact of value enhancement on Decision Generation can be analyzed using the two stage model of decision production. As a result of value enhancement, the levels of information accuracy and / or coverage input into the Decision Generation process change. If the quantities of decision labor and capital (L_2 and K_2) are not altered, value enhancement affects decision accuracy and coverage leading to a different level of gross payoff. Alternatively, the levels of decision labor and capital may be altered either to substitute for information accuracy and coverage to keep decision accuracy and coverage at constant levels, or to attain new levels of decision accuracy and coverage.

Since value enhancement affects both Information Generation and Decision Generation, the comparison of alternative MIS must be based on the overall decision production process. Based on Postulate 2, Section 3.3, a criterion of relative productivity can be proposed for the comparison of alternative MIS:

Relative Productivity of Decision Production: Given two MIS, A and B, for a DU, A is considered more productive than B if it results in a higher net payoff. That is, the productivity of an MIS is jointly determined by the gross payoff and total cost of decision production. Therefore, an existing MIS may be replaced by a more costly one if the gain in gross payoff due to the new system more than compensates for the increase in decision production cost.

Two weaker, though consistent, versions of the criterion of relative productivity can be derived from Postulates 2A and 2B (Section 3.3):

Relative Economic Efficiency of Decision Production: Given two MIS, A and B, for a DU, A is considered economically more efficient than B if it results in a lower total cost for the same amount of gross payoff. That is, efficiency, or doing the thing right, can be operationalized by minimizing the cost of input resources, *ceteris paribus*. (As with cost reduction, two more criteria of relative efficiency, technical and allocative efficiency of decision production, can be proposed for value enhancement in a similar manner.)

The criterion of relative economic efficiency is often used to justify the increased level of automation for structured decision making. For example, computer based systems are used to detect reorder points for inventory items, and suggest economic order quantities to inventory managers thus reducing the cost of decision making.

Relative Effectiveness of Decision Production: Given two MIS, A and B, for a DU, A is considered more effective than B if it results in a higher gross payoff for the same amount of total cost. Thus effectiveness, or doing the right thing, can be operationalized by maximizing gross payoff, *ceteris paribus*.

The criterion of relative effectiveness emphasizes the need to improve decision accuracy and coverage to increase the gross payoff of decision production. One common argument for decision support techniques relate to this criterion. It is often argued in the MIS literature that while traditional data processing techniques fail to improve the effectiveness of decision making, the use of decision support techniques based on modeling and graphics technology can improve decision effectiveness [Keen and Scott Morton 1978, p. 1].

In summary, the criterion of relative productivity of decision production can be used to compare any set of alternative MIS, and thus can be considered as the global rule for MIS evaluation. But the application of this criterion demands maximum effort on the part of the analyst since it requires the estimation of both gross payoff and cost of decision production. All other rules of MIS comparison (including those of cost reduction) are special cases of the criterion of relative productivity. When applicable, the special cases reduce the effort required for MIS evaluation. An analyst may use the criterion of relative productivity and / or its weaker derivatives to establish a weak ordering of any set of alternative MIS for a given DU.

The use of the criteria of MIS evaluation requires the identification and measurement of the inputs and outputs of the decision production model. Procedures to deal with the identification and measurement problems are discussed in the following section.

3.5 Identification and Measurement of Inputs and Outputs

As described in Section 3.2, the Information Generation process involves transforming data into information useful for decision making

using various types of MIS labor and capital. The Decision Generation process, on the other hand, entails making decisions from information using decision labor and capital. Data, information, and decisions as inputs and outputs of production can be measured using the scheme described in Section 3.1. However, each of the input resources (various types of capital and labor) must also be fully specified for MIS evaluation. This section enumerates procedures for the identification and measurement of input resources.

The identification of input resources is based on the definition of the factors of production and the microeconomic assumption of constant technology over the production period. The definition of factors of production implies the following [Berczi 1981]:

1. The combination of factors of production generates products. If an additional quantity of a factor is added to the fixed quantities of other inputs, a higher output should result. For example, the services of data entry clerks are considered an input for the Information Generation process because increasing the hours worked by data entry clerks should lead to higher information accuracy (e.g., if additional hours worked are used for the verification step of card punching operation) and / or coverage (if supplementary labor is used to increase the number of source documents transcribed into auxiliary memory).

2. Factors of production have factor prices. Only economic resources with nonzero prices are candidates for inputs. Air is a resource required for decision production, but is not considered an input. Electricity, on the other hand, has a specific rate, and so may qualify as an input.

3. Factors of production have an identifiable source of supply. Each

input can be traced to a specific origin. For example, each program must have been either internally developed or leased from a software vendor. There can be no software input that does not fall under either category.

4. Factors of production have an identifiable source of demand. An examination should be made to see if a resource is actually used in the decision production process. For example, the services of programmers or system analysts are not considered inputs if no modification or maintenance of any software is carried out during the production period.

5. Factors of production have factor markets. There must be parties willing to buy or sell input resources. For example, computer time can be considered an input because computer time is bought and sold in open markets.

6. Only the factors under the firm's control are of interest. For example, climatic conditions may affect production, but the climate is not considered an input.

The assumption of constant technology implies a second set of guidelines for the identification of input resources:

1. The set of inputs must not change over the production period [Frisch 1956, p.25]. That is, all inputs must be in use throughout the production period. For example, no new hardware is assumed be introduced during the production period.

2. The characteristics of each input must be fully specified [Dano 1966, p.6]. This requires the enumeration of the properties of inputs relevant to the decision production process. For example, all relevant features of a software that may have any effect on decision production should be specified.

3. The assumption of constant technology also implies that the

properties of inputs do not change over the production period [Ferguson 1969, p.60]. For example, if a minicomputer is replaced by a mainframe, it amounts to a change in the input mix.

4. Different grades of a resource should be considered as different inputs [Dano 1966, p.6]. For example, a novice programmer may be distinguished from an expert programmer on the basis of programming experience.

The differentiation of inputs based on these principles may often lead to too many input types, and thus increase the difficulty of MIS evaluation. Two methods can be used to circumvent this problem. First, if a set of inputs vary proportionately for technical reasons, they can be represented as a single input [Dano 1966, p.6]. Second, an average technological specification with associated tolerances may be used for each input type [Cooper 1983a, p.28]. Later if more precision is required, number of input types may be increased by reducing the tolerances imposed on their average characteristics.

For the purpose of measurement of input levels, input resources may be classified along two dimensions: durability and variability. Nondurable inputs are consumed during production and cannot be reused. Durable inputs are not used up, and their services are input into the production process. With respect to variability, inputs are termed fixed or variable. The quantity available of a fixed input is invariant with respect to the quantity of output produced. The quantity of variable input, on the other hand, depends on the level of output produced.

The measurement of variable inputs is relatively straight forward. Variable nondurable inputs are measured in terms of consumption flows per unit time. For example, electricity can be measured in kilowatt

hours. Variable durable inputs are measured in terms of spatial or temporal stock of input used during the production period. All labor inputs, decision makers, computer operators, programmers, etc. are measured in person-hours.

The measurement of fixed inputs is somewhat more complex. Before the fixed capacity of an input is reached, it can be measured in terms of consumption flows (if nondurable), service flows (if durable and temporarily divisible) or portions of stock (if durable and spatially divisible). When the capacity of the fixed input is reached, it is represented as a function parameter or by the shape of the production function [Dano 1966, p.8]. For example, a durable fixed input such as CPU is measured in seconds (temporal division) while primary storage is measured in bytes (spatial division).

The computer configuration has a direct bearing on the complexity of resource measurement. For example, when a dedicated system is used for information generation, estimation of the levels of inputs is relatively simple. However, if multiple MIS share a computer installation, determination of resource utilization by individual MIS becomes difficult. When a centralized computer is operated in a uniprogramming mode, resource utilization by each MIS can be determined by treating the computer as a single resource. Hence a single factor, such as wall clock time or CPU time, can be used as a measure of computer usage [Cushing 1976]. But this method fails if all other factors do not vary proportionally with the single factor chosen. In such cases multiple measures such as CPU time, occupancy of primary storage, direct access storage space, tape mounts, graphic console time, etc. are required [Gladney 1975]. Finally, the measurement of inputs becomes highly

complex in a multiprogrammed, time shared computing environment. A single measure of utilization is ruled out since one MIS may not use all resources at one time, and other MIS may simultaneously use some of the remaining resources. Consequently input levels of each MIS include resources actually used and resources otherwise made unavailable to other MIS [McKell 1979].

The measurement of computer software, on the other hand, is analogous to the measurement of labor inputs in that both can be viewed as providing a set skills to the production process [Cooper 1983a, p.33]. First, different software packages can be classified based on capabilities such as word processing, modeling, statistical ability, etc. This step can be compared to classifying labor into categories such as computer operators, system analysts, programmers and typists. Next, time required to complete a standard task (benchmark) can be used to assess the "skill" level of each software of its class, just as typing speed can be used to appraise skill levels of typists. Finally, software hours of each software type can be used as input measures.

3.6 Summary

The proposed approach of MIS evaluation is discussed in this chapter. An MIS is viewed as part of a decision production process. According to this approach, decisions are produced in much the same way as normal goods and services. The raw materials of decision production are data which go through various conversions during the production process resulting in decisions.

The proposed approach is based on the concept of a decision unit that produces one type of decision repeatedly. It is hypothesized that

the production of decisions in a DU takes place in two stages. First, an MIS converts raw data into information useful for decision making. Next, information is entered into a decision model to produce decisions. At each stage of production, various types of labor and capital are employed to effect the transformation process.

A scheme for the measurement of data, information, and decisions is also introduced in this chapter. The scheme consists of two orthogonal dimensions: accuracy and coverage. Accuracy refers to the extent to which a description contained in data, information, or decisions is in accord with reality, whereas coverage is a measure of how inclusively it describes relevant parts of reality. Procedures for the measurement of both accuracy and coverage are discussed in this chapter.

Two postulates are made about the behavior of a DU to enable the comparison of alternative MIS. Based on these postulates, a criterion of relative productivity is formulated for the comparison of alternative MIS. Several weaker, though consistent, criteria of relative efficiency and effectiveness are also specified. Finally, procedures for the identification and measurement of inputs and outputs are discussed.

The proposed model of decision production can be used to estimate the effects of MIS on firm productivity. As an example of this approach, a decision production model is developed in the next chapter to evaluate fixed reorder cycle inventory control system, and an analytical method is used to derive some preliminary results.

CHAPTER 4
EVALUATION OF FIXED REORDER CYCLE SYSTEMS

A two stage model of decision production was presented in the previous chapter. Based on this model, certain criteria were stated for the comparison of alternative MIS. The purpose of this chapter is to illustrate and validate the decision production model in a specific decision context. The selection of the context is based on two criteria: simplicity and importance of the decision problem. The decision context chosen represents order quantity decision making with an inventory control MIS.

In particular, the two stage decision production model is used to evaluate reorder point inventory control systems for independent demand items. Independent demand items are those items whose demand is created by forces outside the control of the production inventory system under consideration [Buffa and Miller 1979, p.155]. Independent demand items include finished goods, supplies and maintenance items, spare parts, retail and wholesale items.

Reorder point systems are widely used for independent demand items. There are three major types of reorder point systems: (1) Fixed reorder cycle, (2) Fixed reorder quantity, and (3) Optional replenishment system [Buffa and Miller 1979, p.161]. Based on the proposed approach of MIS evaluation, a decision production model for a fixed reorder cycle system is presented in this chapter. Next an analytical approach is used to

evaluate the MIS of a fixed reorder cycle system. Based on a few assumptions, certain properties of the fixed reorder cycle information system are examined in this chapter. However, the results obtained in this chapter are applicable to "good quality" MIS only. To overcome this limitation, a simulation is designed in the next chapter to examine more realistic information systems.

The fixed reorder cycle system is examined here for two cases: (1) Lost sales case, and (2) Backordering case. In the first case, if customer orders are not satisfied from ready stock, customers turn to alternative sources of supply. In the second case, if current stock is not sufficient to meet demand, customer orders can be backordered (usually by incurring additional costs), and filled in near future. The lost sales case is the primary focus of discussion in this chapter. The backordering case is handled by making appropriate modifications to the lost sales case.

This chapter is organized as follows. First, a fixed reorder cycle inventory control system is described in detail in Section 4.1. A summary of prior research on fixed reorder cycle systems is briefly presented next in Section 4.2. A numerical example is used in Section 4.3 to illustrate the decision production process in a fixed reorder cycle system. The gross payoff and decision production functions of a fixed reorder cycle system are presented in Section 4.4. An analytical formulation of the problem is given next in Section 4.5. Using a few assumptions, an analytical solution is obtained for the lost sales case in Section 4.6. The analytical solution is modified in Section 4.7 to take into account the possibility of backordering customer orders. Finally, a summary of this chapter is given in Section 4.8.

4.1. A Fixed Reorder Cycle System

A fixed reorder cycle inventory control system makes decisions on a periodic basis. The MIS of a fixed reorder cycle system provides two types of information to aid decision making: (1) order-up-to point information that indicates future requirement of an item, (2) quantity on record information that represents the availability of an item. At the end of each review period, a tentative order quantity is determined by subtracting quantity on record (q) from the order-up-to point (r); however, the tentative order quantity is set to zero if quantity on record exceeds the order-up-to point:

$$\begin{aligned} \text{Tentative Order quantity} &= r - q && \text{if } r > q, \\ &= 0 && \text{otherwise.} \end{aligned}$$

The tentative order quantity is next reviewed by management to guard against any abnormal order quantity arising out of inaccurate information. Sometimes management may also alter the order quantity based on its judgment and experience.

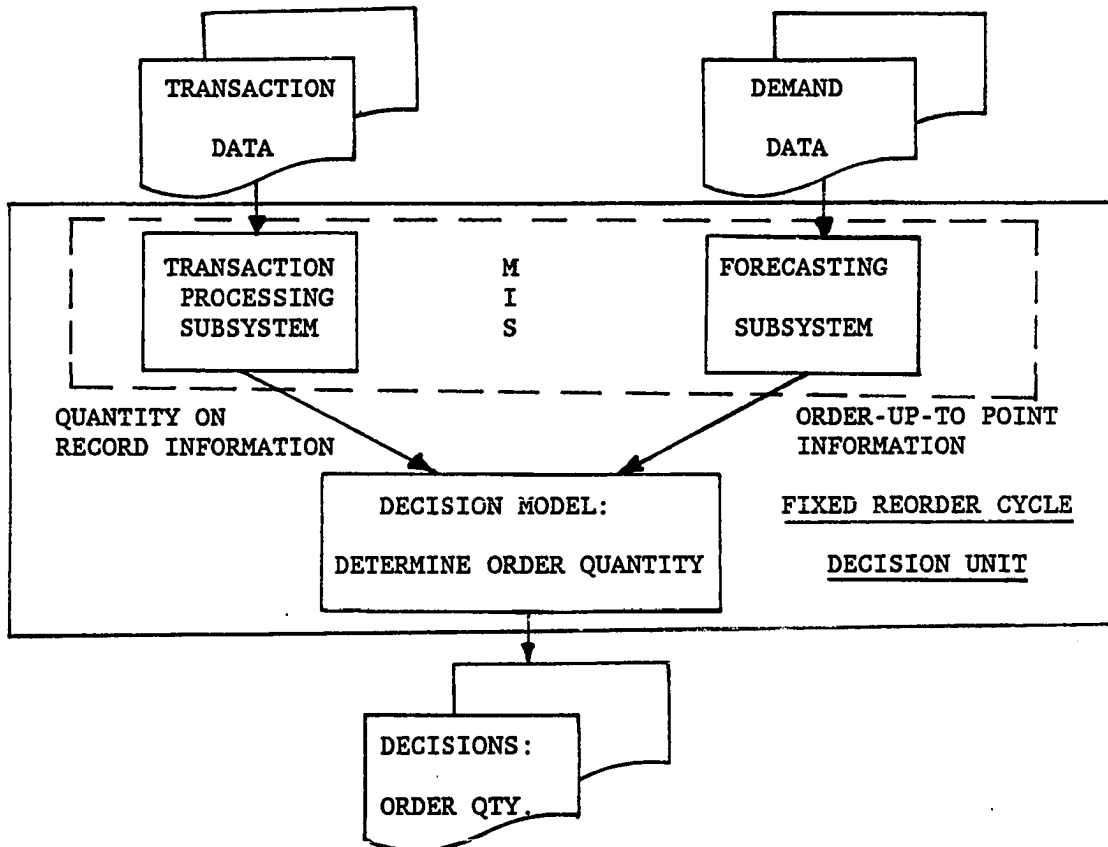
The MIS of a fixed reorder cycle system consists of two subsystems: (1) a transaction processing subsystem that keeps track of quantity on record of each item, and (2) a forecasting subsystem that updates the order-up-to point [Figure 4.1]. Each of these two subsystems is described below.

Transaction Processing Subsystem: This subsystem keeps track of shipments, customer orders and returns, purchase orders, receipts, scrap records, and other transactions. The transactions are recorded in a file, and are used to update quantity on record of each item (Quantity on record = Quantity on hand + Quantity on order - Quantity backordered). Quantity on record of each item is updated prior to making

the order quantity decision.

Since quantity on record of an item constantly changes with time, it is important to report this information in a timely manner. A variety of design alternatives are available to management to control the timeliness of quantity on record information. The major design alternatives can be classified into the following categories: (a) Manual system, (b) Batch system; (c) Deferred on-line system (On-line data entry and batch update), and (d) Fully on-line system. Typically, the timeliness of quantity on record information improves progressively from a manual system to a fully on-line system, although the exact degree of improvement would depend on the detailed specifications of a system.

FIGURE 4.1: A FIXED REORDER CYCLE SYSTEM



In short, the timeliness of quantity on record information can be improved to the maximum level in case of a fully on-line system. Such a system updates quantity on record as soon as a transaction is input into the system. However, even such a system may not make quantity on record information absolutely timely due to human interventions. For example, all transactions may not be recorded as soon as they occur. Moreover, some finite amount of time is taken by management to review and approve the tentative order quantities suggested by the computer system.

Forecasting subsystem: This subsystem determines an order-up-to point based on the total demand during replenishment lead time and review cycle [Hax and Candea 1984, p. 193]. Although the number of factors which can possibly affect the order-up-to point may be enormous, any such factor can be classified into one of two categories: (1) regular factors that generated demand in the past, and would continue to affect demand in the future, and (2) special factors that appear for the first time in affecting total demand [Brown 1977, p. 73]. Routine statistical methods are used to forecast the effect of regular factors, leaving the forecaster free to predict the effect of special factors such as promotional campaigns, competitive actions, economic and demographic variations, changes in consumer tastes and governmental regulations. In summary, an appropriate order-up-to point is selected by combining human judgment with a demand projection based on past demand data.

An important difference between quantity on record and order-up-to point information is that while quantity on record describes the present stock situation, order-up-to point predicts the future stock requirement. Since future requirements can not be based on past demand

data alone, compared to quantity on record information, the timeliness of order-up-to point information is typically not as critical a consideration in designing an appropriate system.

The frequency with which the order-up-to point is updated depends on the stability of demand conditions. If demand conditions change rapidly, the order-up-to point must be updated frequently. However, the order-up-to point need not be modified more than once during a review cycle because it is used only once per cycle in making the order quantity decision.

Before examining the decision production process in a fixed reorder cycle system, it is useful to review prior research in this area. Thus a summary of prior research is given in the next section.

4.2 Prior Research

The inventory control literature provides a stream of research on fixed reorder cycle systems. The objective of this line of research is to formulate procedures for the selection of the optimal order-up-to point and review period. Naddor [1966], for example, studies a variety of fixed reorder cycle systems with both zero and non-zero lead times. In some of the systems he examines the order-up-to point is known, and the optimum review period is to be determined. In other systems, the review period is given, and the optimal order-up-to point is to be found. He also studies systems which have no constraints either on the review period or on the order-up-to point.

Hadley and Whitin [1963, Ch. 5] derive the cost equations of a fixed reorder cycle system for two cases: Poisson and normally distributed demand conditions. They assume that the cost of each backorder is

constant, and is independent of the length for which it exists. The resulting relations for the determination of the two controls, the order-up-to point and review period, however, are too complicated to be of use for routine applications [Hax and Candea 1984, p. 225].

Hax and Candea [1984, p. 226] present an approximate heuristic treatment of the fixed reorder cycle system which yields simpler results. They assume that the demand distribution is stationary and known. They derive expressions for the optimal order-up-to point and review period for both backorder and lost sales cases.

An alternative method for the determination of the order-up-to point is to specify a service level for the inventory item [Buffa and Miller 1979, p. 166]. The underlying principle of this method is that the level of the order-up-to point should provide for reasonable maximum demand during the replenishment lead time plus a review period. For example, in case of a normal demand distribution $N(\mu, \sigma^2)$, a service level of approximately 97 percent would define the order-up-to point as the reasonable maximum demand of $\mu + 2\sigma$.

The focus of the research in the inventory control literature is to formulate procedures for the determination of the order-up-to point and review period, and not on the effects of information on these inventory control systems. As a result, an important assumption of this line of research is that the decision maker has accurate knowledge of the stochastic demand distribution. It is also assumed that the quantity on record information is accurate. Moreover, this stream of research is not concerned with the effects of changing the coverage of the order-up-to point and quantity on record information.

In summary, the inventory control literature does not attempt to

determine the effects of information accuracy and coverage on decision accuracy, or the substitution possibilities between information accuracy and coverage. Similarly, it is not concerned with the impacts of decision accuracy and coverage on net contribution, or the substitution possibilities between decision accuracy and coverage.

The decision production model described in Chapter 3 can be used to bridge the research gap in this area. The decision production process in a fixed reorder cycle system will be discussed in the following sections. However, first a numerical example will be given in Section 4.3 to help illustrate this process in subsequent discussions.

4.3 A Numerical Example

A numerical example is presented to illustrate the decision production process in a fixed reorder cycle system. For simplicity decision making with perfect information is examined in the lost sales case. To simplify the decision problem further the marginal cost of lost sales is assumed to be greater than the marginal cost of carrying additional inventory in the relevant range of decision making.

Although the review period in this section is described in days, it can be interpreted as any other time unit such as hour, week or month. Two cases are considered in this example: (1) Daily decision making, and (2) Decision making every other day.

Case 1: Daily Decision Making

Accurate order quantity decisions made by a fixed reorder cycle system require that the reorder point model used to make the decision exactly replicates reality. That is, accurate order quantity decisions can only be made with perfect knowledge of future demand (to select the

accurate order-up-to point) and accurate quantity on record information. For example, consider a demand stream of an item with fixed lead time of four days given in Figure 4.2. Assume that quantity on record information is accurate and timely. With perfect knowledge of future demand, accurate order quantity decisions can be made using the following rule:

FIGURE 4.2 : ACCURATE ORDER QUANTITY DECISIONS (CASE 1)

I Day	II Opening Stock	End of Day Figures					
		III Actual Demand	IV Stock Out	V Quantity ¹ On Record	Quantities After Decision		
					VI Order Quantity	VII Quantity On Order	VIII Quantity ² On Record
1	2500	631	0	1869	456	456	2325
2	1869	478	0	1847	654	1110	2501
3	1391	402	0	2099	493	1603	2592
4	989	547	0	2045	317	1920	2362
5	442	589	147	1920	585	2505	2505
6	456	456	0	2049	707	2756	2756
7	654	654	0	2102	432	2534	2534
8	493	493	0	2041	?	?	?
9	317	317	0	?	?	?	?
10	585	585	0	?	?	?	?
11	707	707	0	?	?	?	?
12	432	432	0	?	?	?	?

¹Quantity on record information is quantity on record before decision.

²Quantity on record after decision equals accurate order-up-to point.

At the end of t -th day, order the quantity equal to the demand of $(t+5)$ th day. Since the order made on t -th day arrives at the beginning of the $(t+5)$ th day, it can be used to fill demand for $(t+5)$ th day and any subsequent days. A rational manager will always follow the above decision rule because, if the order quantity is greater than the demand on $(t+5)$ th day, inventory carrying cost increases, and if the order quantity is smaller, stockout cost (lost sales) rises.

Figure 4.2 illustrates this example. The starting conditions for the first day for this case are: Quantity on hand = 2500, Quantity on order = 0. The demand on the first day is 631, resulting in 1869 units left at the end of the day. Given perfect knowledge of future demand and quantity on record information, the order quantity for the first day should be equal to the demand 456 of the sixth day. Thus quantity on record after the decision is 2325 units (Column VIII, Figure 4.2). Since, for a nonzero order quantity, quantity on record (after decision) equals order-up-to point, with accurate demand data, the order-up-to point for the first day is 2325. Similarly, the accurate order-up-to point for later days can be found to be 2501, 2592, etc.

Case 2: Decision Making Every Other Day

If the fixed reorder cycle system makes decisions every other day, orders must be placed to cover demand for two consecutive days. Thus the new rule for accurate decisions becomes:

At the end of t -th day, order the quantity equal to the demand of $(t+5)$ th and $(t+6)$ th day. Figure 4.3 uses the demand stream of Figure 4.2 to illustrate the modified reorder point model. At the end of the first day, the modified model accurately predicts the total demand for sixth and seventh day as 1110, and places an order to cover this quantity.

Note that the figures for the even numbered days 2, 4, 6 etc. are given in parentheses to indicate that the decision model does not make use of any information on these days.

FIGURE 4.3 : ACCURATE ORDER QUANTITY DECISIONS (CASE 2)

I Day	II Opening Stock	End of Day Figures					
		III Actual Demand	IV Stock Out	V Quantity On Record	Quantities After Decision		
					VI Order Quantity	VII Quantity On Order	VIII Quantity On Record
1	2500	631	0	1869	1110	1110	2979
2	(1869)	(478)	(0)	(2501)	-	(1110)	(2501)
3	1391	402	0	2099	810	1920	2909
4	(989)	(547)	(0)	(2362)	-	(1920)	(2362)
5	442	589	147	1920	1292	3212	3212
6	(1110)	(456)	(0)	(2756)	-	(2102)	(2756)
7	654	654	0	2102	?	?	?
8	(810)	(493)	(0)	(?)	-	(?)	(?)
9	317	317	0	?	?	?	?
10	(1292)	(585)	(0)	(?)	-	(?)	(?)
11	707	707	0	?	?	?	?
12	(?)	(432)	(0)	(?)	-	(?)	(?)

Although the models depicted in Figures 4.2 and 4.3 are both accurate, they lead to significant differences due to the change in decision frequency:

1. Since decisions are made every other day in Figure 4.3, both accurate order-up-to point and order quantity are higher with the

modified model.

2. Since average quantity on hand increases in Figure 4.3, inventory cost increases, and gross payoff measured in terms of net contribution [Net contribution = Total contribution - Inventory costs] decreases.

In summary, the numerical example above illustrated two decision frequencies in a fixed reorder cycle system with perfect information. Next, the assumption of perfect information is dropped, and a general model of decision production is presented in the following section.

4.4. A Decision Production Model

The evaluation of fixed reorder cycle systems requires full specification of the decision production model. In other words, both the gross payoff and decision production functions should be estimated to evaluate a fixed reorder cycle system. Each of these functions will be discussed in this section.

Gross Payoff Function:

Since the purpose of a fixed reorder cycle system is to maximize net contribution, gross payoff Y is measured in terms of net contribution. A neoclassical production function can be used to relate gross payoff with decision accuracy (D_a) and coverage (D_c):

$$(4.1) Y = f(D_a, D_c).$$

Before examining the gross payoff function given in (4.1), it is important to specify the measurement schemes for both decision accuracy and coverage. Thus these two measurement schemes are described next, followed by a discussion of the properties of the gross payoff function.

Measurement of Decision Coverage: The relevant attribute of decision coverage is temporal detail because all other attributes can be

considered unimportant or fixed during the production period:

* Spatial detail is constant since decisions are made at the individual item level.

* Spatial scope can also be considered fixed if the number of inventory items do not change during the production period.

* Temporal scope is not relevant because alternative MIS are compared across the same time period.

* Timeliness becomes relevant if the time gap between decision making and placing orders is different for different MIS. But this is a simple procedural problem, and is not an important issue for MIS evaluation.

* The temporal detail of decision coverage, however, is considered important because it is directly linked to the review period. A shorter review period allows decisions to be made more frequently, and hence leads to higher temporal detail.

Since the relevant attribute of decision coverage is temporal detail, decision coverage (D_c) can be measured as

$$(4.2) D_c = 1 / T$$

where T represents the review time for the fixed reorder cycle system.

Measurement of Decision Accuracy: The first step in measuring decision accuracy is to identify the relevant components of the error term. The decision alternative chosen by a fixed reorder cycle system may exhibit both random and systematic errors:

* Random errors are caused by two sources: (1) stochastic demand conditions, and (2) errors in quantity on record information.

* The presence of systematic error is borne out by the fact that average shortage (negative deviation) is negligibly small when compared with average excess inventory (positive deviation) thus leading to a

positive bias [Hax and Candea 1984, p.195].

* Since a single decision alternative is chosen for each inventory item during a review period, there is no opportunity for decision conflict.

In summary, the relevant components of the error term of decision alternative are random and systematic errors. Therefore decision accuracy (D_a) can be measured as:

$$(4.3) D_a = 1 - [\text{MSE}(d)]^{1/2} / d$$

where $\text{MSE}(d)$ represents the mean squared error of decision alternative (d) and d is the mean of the accurate decision alternatives.

Properties of the Gross Payoff Function: The numerical example given in Section 4.3 can be used to examine certain properties of the gross payoff function. In particular, the effects of each input (decision accuracy and coverage) on gross payoff and the substitution possibilities between the two inputs will be considered here.

It is expected that an increase (decrease) in decision accuracy has a positive (negative) effect on gross payoff. For example, consider the decision taken on the first day in Figure 4.2. Suppose the order quantity selected is 500 ($D_a < 1.00$) instead of 456 ($D_a = 1.00$). As a result, excess inventory occurs and gross payoff measured in terms of net contribution reduces.

It is also expected that an increase (decrease) in decision coverage has a positive (negative) effect on gross payoff. In the context of the numerical example, the model in Figure 4.2 corresponds to $D_a = 1.00$ and $D_c = 1.00$, whereas the model in Figure 4.3 represents $D_a = 1.00$ and $D_c = 0.50$. However, the latter model leads to excess inventory, and thus reduces gross payoff. Decreasing decision coverage from 1.00 to 0.50

with a constant level of decision accuracy ($D_a = 1.00$), leads to lower gross payoff.

The possibility of substitution between decision accuracy and coverage can also be seen from the numerical example of the previous section. For example, as decision accuracy for the daily decision making case in Figure 4.2 is reduced, gross payoff diminishes due to additional inventory related costs. Thus it is plausible that gross payoff for the daily decision making case ($D_c = 1$) equals that of Figure 4.3 model ($D_a = 1$, $D_c = 0.5$) for a certain level of decision accuracy $D_a < 1$. That is, decision accuracy can be substituted by decision coverage keeping a constant level of gross payoff and vice versa.

The simple nature of the numerical example does not allow a detailed examination of the properties of the gross payoff function. Such an analysis will be performed in a latter section. The remainder of this section is devoted to the discussion of the decision production function.

Decision Production Function:

The purpose of this research is to compare alternative MISs. If alternative MISs are specified in terms of their outputs (information accuracy and coverage), alternative systems can be compared by examining the effects of information accuracy and coverage on the decisions being made. In other words, by specifying alternative MIS in terms of their outputs, the study of the decision production process can be limited to the modeling of the decision generation stage only [see Section 3.2].

It is postulated that the transformation of information into decisions can be characterized by using a neoclassical production function. The following discussion presents the decision generation

function assuming that decision labor and capital are kept at constant levels.

For a given level of decision coverage, decision accuracy can be given as a function of order-up-to point accuracy (R_a) and coverage (R_c) and quantity on record accuracy (Q_a) and coverage (Q_c) [see equation (3.3a)]:

$$(4.4) D_a = g(R_a, R_c, Q_a, Q_c).$$

As is described in Section 3.2, it is expected that the marginal product of each input of (4.4) is positive and diminishing. Moreover, the possibility of input substitution is also allowed. For example, the same level of decision accuracy is likely to result from different combination of R_a and Q_a , ceteris paribus. The remainder of this section describes the measurement schemes for information accuracy and coverage.

Measurement of Information Accuracy: Under ordinary circumstances the quantity on record information generated by the MIS may be subject to an error. The relevant components of the error term can be determined as follows:

* Assuming that the MIS generates a single description of quantity on record information, the conflict component of error can be set equal to zero.

* The existence of a systematic error component is ruled out because any such error is likely to be detected and corrected during periodic physical inventory counting.

* The error in quantity on record information is the result of many factors such as sampling errors, measurement errors, transcription errors, transposition errors, logical errors, etc. [Kleijnen 1980, p.166]. Since the occurrence of these errors is not known a priori, a

random component is used to summarize them.

In short, quantity on record information is assumed here to be subjected to random errors only. Thus the accuracy of quantity on record information (Q_a) can be measured as:

$$(4.5) Q_a = 1 - \sigma_q / q$$

where σ_q is the standard deviation of the error term and q is the mean of the accurate quantity on record information over the production period.

Similarly the order-up-to point information exhibits no conflict if the MIS generates a single description of the order-up-to point. However, order-up-to point information exhibits both systematic and random errors. The presence of random error is caused by stochastic demand conditions, whereas a positive bias is typically created by overestimating the order-up-to point to protect against stockouts [Buffa and Miller 1979, p.164]. Thus accuracy of order-up-to point information can be measured as:

$$(4.6) R_a = 1 - ([MSE(r)]^{1/2} / r)$$

where $MSE(r)$ represents the mean squared error of order-up-to point (r) and r is the mean of accurate order-up-to points.

Measurement of Information Coverage: As is described in Section 4.1, the relevant attribute of coverage of quantity on record information is timeliness. If the age of quantity on record information is denoted by T_q , the coverage of quantity on record information can be given as:

$$(4.7) Q_c = 1 / T_q.$$

The relevant attribute of order-up-to point information was identified in Section 4.1 as the frequency of update (temporal detail). The maximum temporal detail of order-up-to point information occurs if

it is updated in every review period (T). Denoting T_r as the time gap between two updates, order-up-to point coverage can, then, be given as:

$$(4.8) R_c = T / T_r.$$

The description of the measurement schemes for information accuracy and coverage completes the specification of the decision generation model. An analytical formulation of this problem is developed next.

4.5. Analytical Problem Formulation

The evaluation of a fixed reorder cycle system requires the derivation of both the gross payoff and decision generation functions. For this purpose, an analytical formulation of this problem is presented in this section. In particular, a fixed reorder cycle system is considered for finished goods control at the factory level.

As stated in the introduction of this chapter, a fixed reorder cycle system is considered for both lost sales and backordering cases. A common formulation for both these cases is given in this section. The major aspects of the analytical formulation are described below.

1. Demand Distribution: The normal distribution describes the demand functions of finished goods adequately, particularly at the factory level [Buffa and Miller 1979, p.134]. Therefore a normal distribution $[N(\mu, \sigma^2)]$ is chosen for the finished goods demand distribution.

2. Quantity on Record Error Distribution: As is described in Section 4.4, quantity on record information is subject to random error only. The error in quantity on record information is the result of many factors such as sampling errors, measurement errors, transcription errors, transposition errors, logical errors, etc. Assuming that positive deviations due to these errors are just as likely as negative

deviations, the error term can be approximated by a normal distribution with zero mean [Kmenta 1971, p.90].

There is an additional restriction on the quantity on record information for the lost sales case (no backordering allowed). Since both quantity on hand and quantity on order can not be negative, quantity on record of an item must not be negative for the lost sales case. However, a perfect normal distribution for the error term may result in negative values for the quantity on record information. For example, if q_i is the accurate quantity on record, and the error term e_{qi} is assumed to be $N(0, \sigma_q^2)$, then the i -th description of quantity on record given by the MIS, denoted by q'_i , may become negative for large negative values of e_{qi} :

$$(4.9) \quad q'_i = q_i + e_{qi}.$$

Therefore, for the lost sales case, a constrained normal distribution should be used for the error term such that $q'_i \geq 0$.

3. Order-up-to Point Error Distribution: The order-up-to point information exhibits both random and systematic errors [Section 4.4]. In addition to the typical data processing errors such as measurement errors, logical errors, transcription errors, transposition errors, sampling errors, the order-up-to point information is subjected to errors arising out of the limitations of the forecasting model and forecaster's judgment. Thus order-up-to point error can also be approximated by a normal distribution.

The order-up-to point information, however, can not be negative for both the lost sales and backordering cases. But a perfectly normal distribution for the error term may also result in negative values for the order-up-to point information. For example, if r_i represents the

i -th accurate order-up-to point, and the error term e_{ri} is assumed to be $N(\mu_r, \sigma_r^2)$ with $\mu_r > 0$, then the i -th description of the order-up-to point information given by the MIS, denoted by r'_i , may become negative for large values of e_{ri} :

$$(4.10) \quad r'_i = r_i + e_{ri}.$$

Since the order-up-to point must not be negative, the error term for the order-up-to point information should also be represented by a constrained normal distribution.

4. Gross Payoff: Gross payoff is measured in terms of net contribution, where net contribution is the total contribution minus relevant inventory costs. In general, inventory costs are classified into three categories: ordering cost, holding cost, and stockout cost. The ordering cost represents the cost of making order quantity decisions. The process of ordering entails keeping inventory records, making necessary computations, making decisions, and communicating the order quantity to the supplier on time. In short, ordering cost is part of the decision production cost, and is not relevant for the computation of gross payoff.

The two other cost components, holding cost and stockout cost, arise out of the imbalance between the demand and supply of an item. Costs associated with holding inventory are due to storage and handling, property taxes, insurance, spoilage, obsolescence, pilferage, and capital requirements. Costs incurred due to stockout, on the other hand, include penalty on late delivery, cost of rush orders, overtime payments, and loss of good will. Both holding and stockout costs are computed based on a cost function taken from Holt, Modigliani, Muth, and Simon (HMMS) [1960]. HMMS estimated a quadratic cost function for a

paint factory. The quadratic cost function proposed by HMMS is considered more realistic than alternative cost structures such as linear cost structure [Hax and Candea 1984, p.88], and has been extensively used by researchers of production and inventory control systems [see Buffa and Taubert 1972, Ch.6].

Following HMMS, the expression for net contribution for the t -th day N_t can be given by:

$$(4.11) N_t = C_0 X_t - C_1 (I_t - X_t - C_2)^2$$

where C_0 , C_1 , C_2 are positive constants, X_t represents demand on the t -th day and I_t denotes the level of inventory at the beginning of the t -th day. Note that $(I_t - X_t)$ represents the level of net inventory, and C_2 denotes the optimal net inventory level. When actual net inventory $(I_t - X_t)$ deviates from the optimal inventory C_2 , net contribution decreases at an increasing rate.

Since all other variables in the decision production model is defined in the range $[0,1]$, the gross payoff (Y_t) will be measured by normalizing its net contribution (N_t) with maximum net contribution possible (N_{\max}):

$$(4.12) Y_t = N_t / N_{\max};$$

where N_{\max} corresponds to an accurate decision model for a daily decision making case.

The analytical problem formulation given in this section can be used to evaluate a fixed reorder cycle system. In the next two sections, an analytical solution for this problem will be discussed. In Section 4.6, an analytical solution will be given for the lost sales case (no backordering), and in Section 4.7 a similar solution will be presented for the backordering case.

4.6. An Analytical Solution for the Lost Sales Case

The analytical method of solution becomes intractable for a realistic fixed reorder cycle system. However, an approximate solution of the analytical problem can be obtained using certain assumptions. Despite its limited applicability, the approximate solution provides some important insights into the fixed reorder cycle system. For example, this solution can be used to estimate the effects of increasing the accuracy of order-up-to point and quantity on record information on decision accuracy. Similarly, the relationship between decision accuracy and gross payoff can also be examined using this solution.

In this section, an analytical solution for the lost sales case is presented in three parts. First, the decision generation function (4.4) is derived, and the properties of the resultant function are examined. Next, the same procedure is repeated for the gross payoff function. Finally, a numerical example is used to illustrate the analytical solution obtained below. See Table 4.1 for a list of symbols used.

Decision Generation Function:

A few assumptions are made to derive the decision generation function for the lost sales case. The analytical method becomes intractable if one or more of the following assumptions are violated.

Assumption 1. It is assumed that the order-up-to point information is generated as often as order quantity decisions are made. In other words,

$$(4.13) D_C = R_C.$$

Assumption 2. The order quantity is assumed to be non-zero. This assumption rules out the possibility that sometimes no order may be placed to reduce excessive inventory build up.

TABLE 4.1: LIST OF FREQUENTLY USED SYMBOLS

Symbol	Explanation	Definition
N_t	Net contribution for t-th day	Equation 4.11
N_{\max}	Maximum net contribution possible	Section 4.5
Y	Gross payoff	Equation 4.12
D_a	Decision accuracy	Equation 4.3
D_c	Decision coverage	Equation 4.2
Q_a	Quantity on record accuracy	Equation 4.5
Q_c	Quantity on record coverage	Equation 4.7
R_a	Order-up-to point accuracy	Equation 4.6
R_c	Order-up-to point coverage	Equation 4.8
X_t	Demand on t-th day	Section 4.5
μ	Mean of demand distribution	Section 4.5
σ	Standard deviation of demand	Section 4.5
e_{qi}	Quantity on record error	Equation 4.9
σ_q	Standard deviation of e_{qi}	Section 4.5
e_{ri}	Order-up-to point error	Equation 4.10
μ_r	Mean of e_{ri} distribution	Section 4.5
σ_r	Standard deviation of e_{ri}	Section 4.5
d	Accurate order quantity decision	Equation 4.3
r	Accurate order-up-to point	Equation 4.5
q	Accurate quantity on record	Equation 4.6
T	Review period	Equation 4.2
L	Lead time	Section 4.6
I_t	Opening inventory on t-th day	Section 4.5
C_0, C_1, C_2	Constants	Section 4.5

Assumption 3. The age of quantity on record information is assumed to be zero. That is, the quantity on record information is assumed to be absolutely current when order quantity decisions are made. The net effect of these two assumptions is to fix the levels of Q_c and R_c in (4.4) at constant levels. As a result, the decision generation function (4.4) reduces to the following:

$$(4.14) D_a = g(R_a, Q_a).$$

Assumption 4. Quantity on record information error is assumed to be perfectly normally distributed. This assumption allows only high values of quantity on record accuracy because at low levels of accuracy, a perfect normal error distribution may lead to negative values of quantity on record [section 4.5].

Assumption 5. Order-up-to point error distribution is also assumed to be perfectly normal. This assumption also restricts the order-up-to point accuracy to high values only.

For a non-zero order quantity, the order quantity decision can be made by subtracting quantity on record from the order-up-to point. That is, if decision error is denoted by e_{di} , then the following relation holds:

$$(4.15) e_{di} = e_{ri} - e_{qi}.$$

Since e_{ri} and e_{qi} are independently and normally distributed, e_{di} must also be normally distributed. In particular, since $E[e_{ri}] = \mu_r$ with $\mu_r > 0$, and $E[e_{qi}] = 0$,

$$(4.16) e_{di} = N(\mu_r, \sigma_r^2 + \sigma_q^2).$$

That is, the mean squared error $MSE(d)$ is equal to $\mu_r^2 + \sigma_r^2 + \sigma_q^2$.

In order to express decision accuracy (D_a) in terms of quantity on record accuracy (Q_a) and order-up-to point accuracy (R_a), the accurate

magnitude of each of the information states in (4.14) must be known. Let d denote the accurate order quantity. Then from (4.3) and (4.16), and since with accurate decisions, $e_{di} = 0$, decision accuracy D_a can be expressed as:

$$(4.17) D_a = 1 - \frac{\sqrt{\mu_r^2 + \sigma_r^2 + \sigma_q^2}}{d}$$

As before let r and q represent the accurate order-up-to point and quantity on record information. Then Q_a and R_a can be given from (4.5) and (4.6) as follows:

$$(4.18) Q_a = 1 - \sigma_q/q$$

$$(4.19) R_a = 1 - \frac{\sqrt{\mu_r^2 + \sigma_r^2}}{r}$$

Substituting $\mu_r^2 + \sigma_r^2 = r^2 (1 - R_a)^2$ from (4.19) and $\sigma_q^2 = q^2 (1 - Q_a)^2$ from (4.18) in (4.17), the decision generation function can be given as:

$$(4.20) D_a = 1 - \frac{[r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{1/2}}{d}$$

Equation (4.20) can be used to examine the properties of the decision generation function. In this section, three important properties of the decision generation function will be studied: (1) Marginal Products of Inputs, (2) Elasticity of Substitution, and (3) Returns to Scale.

(1) Marginal Products of Inputs: The marginal product of an input (first order partial derivative of the production function) represents the rate of increase of the output for a small increase in the input. For this purpose, the first and second order partial derivatives of (4.20) with respect of Q_a and R_a are given below [See Appendix A1.1 for

the derivations of the following equations]:

$$(4.21) \quad \frac{\delta D_a}{\delta Q_a} = \frac{q^2}{d} \frac{1 - Q_a}{[r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{1/2}} > 0,$$

$$(4.22) \quad \frac{\delta^2 D_a}{\delta Q_a^2} = - \frac{q^2}{d} \frac{r^2 (1 - R_a)^2}{[r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} < 0,$$

$$(4.23) \quad \frac{\delta D_a}{\delta R_a} = \frac{r^2}{d} \frac{1 - R_a}{[r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{1/2}} > 0,$$

$$(4.24) \quad \frac{\delta^2 D_a}{\delta R_a^2} = - \frac{r^2}{d} \frac{q^2 (1 - Q_a)^2}{[r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} < 0.$$

As demonstrated above, the marginal product of each input (Q_a and R_a) is positive but diminishing. Thus increasing quantity on record accuracy, for example, leads to an increase in decision accuracy although at a diminishing rate. This result corresponds to a neoclassical production process.

(2) Elasticity of Substitution: The ease with which one input can be substituted for another is given by the elasticity of substitution of a production function. The elasticity of substitution is defined as the proportionate rate of change of input ratio divided by the proportionate rate of change of technical rate of substitution. Thus elasticity of substitution (S) can be written as [Intriligator 1978, p.265]:

$$(4.25) \quad S = \frac{d \ln m}{d \ln n}$$

where m and n are defined as follows:

$$(4.26) \quad m = \frac{Q_a}{R_a},$$

$$(4.27) \quad n = \frac{MP(R_a)}{MP(Q_a)} = \frac{r^2 (1 - R_a)}{q^2 (1 - Q_a)};$$

where $MP(R_a)$ and $MP(Q_a)$ denote marginal products of R_a and Q_a given in equations (4.21) and (4.23) respectively. It can be shown that the elasticity of substitution (S) of (4.20) is given by (4.28) below [See

Appendix A1.2 for derivation]:

$$(4.28) \quad S = \frac{\frac{q^2(1-Q_a)}{R_a} + \frac{r^2(1-R_a)}{Q_a}}{\frac{q^2(1-Q_a)}{1-R_a} + \frac{r^2(1-R_a)}{1-Q_a}} \geq 0.$$

Since the maximum values of Q_a and R_a can not exceed one, elasticity of substitution in (4.28) can not be a negative number. For the input ray $Q_a = R_a$, it can be shown that $S = 1/R_a - 1$. In other words, there is an inverse relationship between the level of information accuracy and elasticity of substitution along this input ray.

(3) Returns to Scale: When all the inputs of a production function are increased by a certain proportion, the output may increase

- (i) at the same rate (constant returns to scale), or
- (ii) less than proportionately (diminishing returns to scale), or
- (iii) more than proportionately (increasing returns to scale).

That is, at the point (Q_a, R_a) , the production function (4.14) $D = g(R_a, Q_a)$ exhibits constant, decreasing or increasing returns to scale if

$$(4.29) \quad g(\lambda Q_a, \lambda R_a) =, < \text{ or } > \lambda g(Q_a, R_a), \text{ for all } \lambda > 1.$$

The returns to scale phenomenon for the production function (4.20) can be determined in the following way [See Appendix A1.3 for details]. Let D'_a correspond to the input combination (R'_a, Q'_a) where $R'_a = \lambda R_a$ and $Q'_a = \lambda Q_a$. Then the decision generation function (4.20) exhibits increasing returns to scale if and only if

$$(4.30) \quad D'_a > \lambda D_a.$$

Equation (4.30) is equivalent to:

$$(4.31) \quad 1 - [r^2(1 - \lambda R_a)^2 + q^2(1 - \lambda Q_a)^2]^{1/2} / d > \lambda - \lambda [r^2(1 - R_a)^2 + q^2(1 - Q_a)^2]^{1/2} / d.$$

Upon simplification (4.31) reduces to:

$$(4.32) \quad \{\lambda^2 r^2 (1-R_a)^2 + \lambda^2 q^2 (1-Q_a)^2\}^{1/2} - (\lambda-1)d \\ > \{[\lambda r(1-R_a) - r(\lambda-1)]^2 + [\lambda q(1-Q_a) - q(\lambda-1)]^2\}^{1/2}.$$

To simplify notation, let

$$(4.33) \quad \Sigma_r = \lambda r (1-R_a), \\ \Sigma_q = \lambda q (1-Q_a), \\ D = (\lambda-1) d, \\ R = (\lambda-1) r, \\ Q = (\lambda-1) q;$$

where $\Sigma_r, \Sigma_q, D, R, Q > 0$ and $D < R > Q$. Using modified notation, the condition for increasing returns to scale (4.32) can be written as:

$$(\Sigma_r^2 + \Sigma_q^2)^{1/2} - D > [(\Sigma_r - R)^2 + (\Sigma_q - Q)^2]^{1/2} \\ \text{or } D (\Sigma_r^2 + \Sigma_q^2)^{1/2} < \Sigma_r R + \Sigma_q Q - C^2;$$

where $C = (R^2 + Q^2 - D^2)/2 > 0$. Upon simplification, the condition for increasing returns to scale becomes:

$$(4.34) \quad \Sigma_r^2 (R^2 - D^2) + C^2 + \Sigma_q^2 (Q^2 - D^2) \\ + \Sigma_r R (\Sigma_q Q - 2C) + \Sigma_q Q (\Sigma_r R - 2C) > 0.$$

Similarly, it can be shown that the decision generation function (4.20) exhibits constant returns to scale if $Z = 0$ and decreasing returns to scale if $Z < 0$ respectively, where

$$(4.35) \quad Z = \Sigma_r^2 (R^2 - D^2) + C^2 + \Sigma_q^2 (Q^2 - D^2) \\ + \Sigma_r R (\Sigma_q Q - 2C) + \Sigma_q Q (\Sigma_r R - 2C).$$

Since $R > D$, the first two terms in (4.34) must be positive. The third term also becomes positive if the review period (T) is smaller than the replenishment lead time (L) such that $Q > D$. Similarly, the last two terms become positive if the following inequalities are satisfied [See Appendix A1.3]:

$$(4.36) \quad R_a < 1 - \frac{2Q}{(R+r)} \quad \text{and} \quad Q_a < 1 - \frac{2R}{(Q+q)}.$$

Since both r and q are large compared to Q and R , both inequalities in (4.36) are likely to be true for most values of R_a and Q_a except when the values of R_a and Q_a are very high. In summary, the likelihood of increasing returns to scale for the decision generation function (4.20) is high for low values of Q_a and R_a .

An intuitive explanation is possible for the increasing returns to scale property of the decision generation function. This explanation is best understood by comparing the lost sales case with the backordering case. The increasing returns to scale for the lost sales case is made possible at the expense of the gross payoff realized. To prevent lost sales, managers must stock more inventory compared to the backordering case even for accurate order quantity decisions. As a result, gross payoff for accurate decisions in the lost sales case is less than or equal to gross payoff for the backordering case. But this condition changes if backordering is allowed, and the likelihood of decreasing returns to scale becomes very high (Section 4.7).

In sum, except for increasing returns to scale, the properties of the decision generation function tend to support the neoclassical view of the decision production process. The properties of the gross payoff function will be studied in the next subsection.

Gross Payoff Function

First the gross payoff of an MIS will be determined as a function of decision accuracy (D_a):

$$(4.37) Y = f(D_a)$$

for two levels of decision coverage (D_c): $D_c = 1$ and $D_c = 0.5$. Next the gross payoff functions for the two levels of decision coverage (D_c) can be used to study the overall gross payoff function (4.1).

Case 1: Review Cycle = 1 day ($D_c = 1$)

In case of $D_c = 1$, accurate decisions refer to the order quantities a rational manager would select on a daily basis when his knowledge of reality is accurate [see Section 3.1]. With accurate knowledge of demand and inventory position, a rational manager would select order quantities to keep the net inventory level ($I_t - X_t$) equal to the optimal inventory level C_2 such that net contribution is maximized [see equation (4.11)]. That is, if the starting inventory level is different from C_2 , the inventory manager would first bring it to the level C_2 , and thereafter select order quantities equal to the daily demand. As a result, the maximum expected net contribution N_{\max} would be:

$$(4.38) N_{\max} = C_0 * \mu.$$

In reality managers neither know the optimal inventory level C_2 nor the daily demand X_t . However, managers know that the opportunity cost of lost sales is much higher compared to the cost of carrying additional inventory. Thus it is expected that managers would systematically overstock inventory to avoid lost sales. As a result, order quantity decisions taken by management would have both systematic and random error components.

Since decision error on t -th day, e_{dt} , is $(I_t - X_t - C_2)$, net contribution for an MIS can be derived from the following expression:

$$(4.39) N_t = C_0 * X_t - C_1 (e_{dt})^2;$$

where e_{dt} has the following distribution [see equation (4.16)]:

$$(4.40) e_{dt} \sim N (\mu_r, \sigma_q^2 + \sigma_r^2).$$

Thus the expected value of net contribution is given by

$$(4.41) E(N_t) = C_0 * \mu - C_1 (\mu_r^2 + \sigma_q^2 + \sigma_r^2).$$

Using (4.17) equation (4.41) can be written as:

$$(4.42) E(N_t) = C_0 * \mu - C_1 d^2 (1 - D_a)^2.$$

The expected value of gross payoff (Y_t) is calculated by dividing (4.42)

by $N_{\max} = C_0 * \mu$:

$$(4.43) E(Y_t) = 1 - \frac{C_1 d^2 (1 - D_a)^2}{C_0 * \mu}.$$

The effect of decision accuracy on gross payoff can be studied from the first and second order derivatives of (4.43) with respect to D_a :

$$(4.44) \frac{d[E(Y_t)]}{dD_a} = \frac{2C_1 d^2 (1 - D_a)}{C_0 * \mu} > 0;$$

$$(4.45) \frac{d^2[E(Y_t)]}{dD_a^2} = -\frac{2C_1 d^2}{C_0 * \mu} < 0.$$

Therefore, the gross payoff increases with an increase in decision accuracy although at a diminishing rate.

Case 2: Review cycle = 2 days or $D_c = 0.5$.

First consider the implications of accurate decisions. Since the marginal cost of lost sales is greater than the marginal cost of excess inventory, a rational manager with perfect knowledge of demand and inventory level would prevent lost sales by ensuring that the inventory at the starting of the review period (I_t) does not fall short of the total demand of the review period ($X_1 + X_2$). This results in excess inventory at the end of the first day of the review period. But is it possible that an accurate decision might lead to excess inventory at the end of the review period?

The HMMS model (see equation 4.11) implies that for low value of demand on the second day (X_2), an optimal order quantity decision may lead to excess inventory at the end of the review period. To see this, consider the decision problem for a rational manager: choose an order

quantity $X_1 + X_2 + \epsilon_t$ (where $\epsilon_t \geq 0$) so as to maximize the net contribution for the review period:

$$(4.46) \text{ Maximize } C_0 (X_1 + X_2) - C_1 (\epsilon_t + X_2 - C_2)^2 - C_1 (\epsilon_t - C_2)^2$$

Subject to $\epsilon_t \geq 0$.

Applying Kuhn-Tucker conditions the optimal value of ϵ_t is given as:

$$(4.47) \epsilon_t = C_2 - X_2/2 \quad \text{if } X_2 < 2 C_2 \\ = 0 \quad \text{if } X_2 \geq 2 C_2.$$

From (4.47) it is clear that a rational manager would overstock inventory if X_2 is low. Moreover, the value of net contribution depends on the relative magnitudes of X_2 and C_2 . However, the upper and lower bounds of net contribution can be determined from (4.47) without making any further assumptions. The lower value of net contribution occurs if $X_2 \geq 2 C_2$ or $\epsilon_t = 0$. To calculate the corresponding expected value of gross payoff (Y_{low}), substitute $\epsilon_t = 0$ in the objective function in (4.46) to obtain:

$$\text{Net Contribution} = C_0 (X_1 + X_2) - C_1 (X_2 - C_2)^2 - C_1 (C_2)^2.$$

Since $X_1 \sim N(\mu, \sigma^2)$, the expected value of net contribution is given by:

$$E(\text{Net Contribution}) = 2C_0\mu - C_1 (\mu^2 + \sigma^2 + 2C_2^2 - 2C_2\mu).$$

Since this expression gives the expected net contribution for two days, the expected net contribution per day can be given by:

$$E(\text{Net contribution}) / \text{day} = C_0\mu - \frac{1}{2} C_1 (\mu^2 + \sigma^2 + 2C_2^2 - 2C_2\mu).$$

Finally, the expected lower bound of gross payoff $E(Y_{low})$ can be calculated by dividing the above expression by $N_{max} = C_0\mu$.

$$(4.48) E(Y_{low}) = 1 - \frac{C_1 (\mu^2 + \sigma^2 + 2C_2^2 - 2C_2\mu)}{2C_0 * \mu}.$$

Similarly, the upper value of net contribution occurs if $X_2 < 2 C_2$, and the resultant expected value of gross payoff can be given as

$$(4.49) \ E(Y_{\text{high}}) = 1 - \frac{C_1(\mu^2 + \sigma^2)}{4C_0 * \mu};$$

where it can be shown that $E(Y_{\text{high}}) > E(Y_{\text{low}})$ for any value of C_2 . The important point, however, is that $E(Y_{\text{high}}) < 1$. That is, irrespective of the relative magnitudes of X_2 and C_2 , the expected gross payoff for the accurate decision model in this case ($D_c = 0.5$) is less than that of the daily decision making case ($D_c = 1$). In other words, the level of gross payoff increases with an increase of decision coverage from 0.5 to 1 for a constant value of decision accuracy of unity. This result supports the hypothesis that the marginal product of decision coverage is positive.

Numerical Example

The numerical example from Section 4.3 can be used to illustrate the properties of the production function (4.20). The example in Section 4.3 was based on a fixed lead time $L = 4$, and a normal demand distribution with average demand $\mu = 524$. For the sake of simplicity, assume that $\epsilon_t = 0$. Then, for example, for $T=6$, the accurate magnitudes of order-up-to point, quantity on record, and order quantity can be calculated as:

$$q = L * \mu = 2096$$

$$r = (L + T) * \mu = 5240$$

$$d = T * \mu = 3144.$$

Thus the decision generation function (4.20) can be written as:

$$(4.50) \ D_a = 1 - [27.46 (1 - R_a)^2 + 4.39 (1 - Q_a)^2]^{1/2} / 3.14.$$

From equations (4.24) through (4.27) it can be seen that the marginal products of (4.50) are positive but diminishing. The elasticity of substitution of (4.50) can also be obtained from equation (4.28). Similarly, the condition for increasing returns to scale can be derived from (4.34) with $\lambda = 1.1$:

$$(4.51) \quad 5838 (1 - R_a)^2 + 12 - 291.9 (1 - Q_a)^2 \\ + 796 (1 - R_a)(1 - 1.83 Q_a) + 1353 (1 - Q_a)(1 - 1.08 R_a) > 0.$$

As stated earlier, the first two terms of (4.51) are positive and the third term is negative (since $L < T$). The last two terms remain positive if $Q_a < 0.546$ and $R_a < 0.926$. However, the relative magnitudes of these terms are such that the inequality (4.51) remains true for most input combinations. Table 4.2 illustrates the values of the elasticity of substitution and returns to scale for different input combinations.

TABLE 4.2: PROPERTIES OF THE DECISION GENERATION FUNCTION

LOST SALES CASE

Quantity on Record Accuracy	Order-up-to Point Accuracy	Elasticity of Substitution	Returns to Scale
0.4	0.4	1.5	Increasing
0.5	0.5	1.0	Increasing
0.6	0.6	0.66	Increasing
0.4	0.6	1.28	Increasing
0.6	0.4	0.72	Increasing

The lost sales case discussed above provides support to the neoclassical view of decision production. A similar analysis of the backordering is presented in the following section.

4.7. An Analytical Solution for the Backordering Case

In the backordering case, if demand exceeds stock on hand, customer orders can be fulfilled at a later date. The analytical solution from the previous section can be adapted for the backordering case with minor modifications. In this section, the decision generation and gross payoff functions are discussed for the backordering case, and a numerical example is used to contrast this case from the lost sales case.

Decision Production Function

The derivation of the decision generation function for the backordering case requires a slight modification of Assumption 4 of the lost sales case. With backordering allowed, quantity on record information is allowed to have negative values provided the sum of quantity on hand and quantity on order remains positive. The decision generation function itself does not require any modification. However, the properties of the decision generation function (4.20) may change because the relative magnitudes of accurate order quantity, order-up-to point, and quantity on record information may change for the backordering case. A numerical example will be used at the end of this section to illustrate this point.

Gross Payoff Function

As before, the gross payoff function for the backordering case will be considered for two levels of decision coverage: $D_c = 1$ (daily decision making) and $D_c = 0.5$ (decision making on alternate days). Each of these cases is discussed below.

The accurate decision model for the daily decision making case does not change if backordering is allowed. With accurate knowledge of demand and inventory position, a rational manager would continue to select order quantities to keep the net inventory $(I_t - X_t)$ equal to the optimal inventory level C_2 such that net contribution is maximized. For daily decision making, therefore, the gross payoff expression (4.45) for the lost sales case is equally applicable for the backordering case. The gross payoff in (4.45), it may be recalled, increases with decision accuracy although at a diminishing rate.

The effect of backordering, however, can be seen for the review

period of two days. With backordering allowed, an accurate decision model would imply that the net inventory $(I_t - X_t)$ on the first day of the review period exceeds the optimal inventory C_2 by half the demand on the second day, since any other decision alternative would reduce the level of net contribution. As a result, net inventory on the second day falls short of the optimal level by the same amount. Thus the gross payoff for the accurate decision model can be given as:

$$(4.52) E(Y_t) = 1 - \frac{C_1(\mu^2 + \sigma^2)}{4C_0 * \mu};$$

where $E(Y_t)$ equals $E(Y_{high})$ from (4.49). In other words, due to the removal of the lost sales constraint, the level of net contribution for the backordering case attains the upper bound for the lost sales case.

As the level of decision accuracy reduces for this case, stockouts and excess inventory would increase. As a result, the level of gross payoff would diminish. The gross payoff of a realistic MIS can be determined by estimating the extent of deviations of net inventory from the optimal level. The level of positive deviation on the first day is the sum of half the demand on the second day $(X_{2t}/2)$ $[N(\mu/2, \sigma^2/4)]$ and decision error e_{dt} $[N(\mu_r, \sigma_r^2 + \sigma_q^2)]$. The amount of negative deviation on the second day, on the other hand, is the difference between half the demand on the second day $X_{2t}/2$ and decision error e_{dt} . Thus the expected net contribution per day can be calculated as follows:

$$(4.53) E(N_t) = C_0 * \mu - C_1 [(\mu^2 + \sigma^2)/4 + \mu_r^2 + \sigma_r^2 + \sigma_q^2].$$

The expected gross payoff can be calculated by dividing (4.53) with $N_{max} = C_0 * \mu$:

$$(4.54) E(Y_t) = 1 - C_1 \frac{(\mu^2 + \sigma^2)/4 + \mu_r^2 + \sigma_r^2 + \sigma_q^2}{C_0 * \mu}.$$

Using (4.17), equation (4.54) can be written as:

$$(4.55) E(Y_t) = 1 - C_1 \frac{(\mu^2 + \sigma^2)/4 + d^2(1 - D_a)^2}{C_0 * \mu}.$$

From equation (4.55) it can be shown that the first and second order derivatives of gross payoff with respect to decision accuracy are positive and negative respectively. Thus an increase in decision accuracy causes an increase in gross payoff although at a diminishing rate.

It can also be seen from (4.45) and (4.55) that for the same level of decision accuracy, daily decision making ($D_c = 1$) results in a higher level of gross payoff than decision making every other day ($D_c = 0.5$). In other words, an increase in decision coverage for a given level of decision accuracy has a positive effect on gross payoff.

Numerical Example

The numerical example used in the previous section can be easily modified to illustrate the decision generation function for the backordering case. With backordering allowed, the accurate magnitudes of order-up-to point and quantity on record information change. For a review period of $T = 6$, for example, it can be shown that: $q = 1106$ and $r = 4250$ [Appendix A1.4]. The resulting function can be given as:

$$(4.56) D_a = 1 - [18.06(1 - R_a)^2 + 1.22(1 - Q_a)^2]^{1/2}/3.14.$$

It can also be shown that (4.56) is likely to exhibit increasing returns to scale for most input combinations.

The discussion in this chapter has thus far dealt with off the shelf products. Finally, consider the case of an industrial product for which customers do not receive immediate delivery. For example, let the supply lead time be $l = 4$ days. With a supply lead time of four days, orders

for four days can be backordered without incurring any penalty. As a result, the accurate magnitude of order-up-to point changes to: $r = 2154$. Thus the decision generation function can be given by (4.57) which is likely to exhibit decreasing returns to scale [Figure 4.5]:

$$(4.57) D_a = 1 - [4.64 (1 - R_a)^2 + 1.22 (1 - Q_a)^2]^{1/2} / 3.14.$$

FIGURE 4.6: PROPERTIES OF THE DECISION GENERATION FUNCTION

BACKORDERING CASE

Quantity on Record Accuracy	Order-up-to Point Accuracy	Elasticity of Substitution	Returns to Scale
0.4	0.4	1.5	Decreasing
0.5	0.5	1.0	Decreasing
0.6	0.6	0.66	Decreasing
0.6	0.4	0.72	Decreasing
0.4	0.6	1.28	Decreasing

In summary, the backordering case also provides support to the neoclassical view of the decision production model. An important difference between this case and the lost sales case is that while the decision generation function exhibits increasing returns to scale in the latter case, it is expected to evidence decreasing returns to scale in the backordering case. The gross payoff function, although not identical for these two cases, is not characterized by any major difference. A summary of this chapter is given in the following section.

4.8. Summary

The purpose of this chapter was to illustrate and validate the neoclassical view of decision production. To attain this goal, first an

analytical formulation for the fixed reorder cycle system was developed. The decision generation function for this system was next obtained in terms of two inputs - order-up-to point and quantity on record accuracy (R_a and Q_a) - using certain assumptions. It was found that an increase in information accuracy (R_a or Q_a) causes an increase in decision accuracy although at a diminishing rate. The elasticity of substitution between the two inputs was also derived. It was also found that the decision generation function is likely to exhibit increasing returns to scale for items requiring immediate delivery and decreasing returns to scale for items that can be backordered.

Moreover, the gross payoff function was also examined to a limited extent. In general, it was found that an increase in decision accuracy results in an increase in gross payoff although at a diminishing rate. Similarly, for any given level of decision accuracy, the level of gross payoff was found to increase for higher values of decision coverage.

The results obtained in this chapter are based on a few simple assumptions. For example, the analytical solution requires that information available be current and relatively accurate. The analytical solution can be considered a first order approximation for "good quality" information systems. In the following chapter, the assumptions made in this chapter will be dropped, and a simulation will be used to extend the analytical results to more realistic information systems.

CHAPTER 5
SIMULATION OF THE FIXED REORDER CYCLE SYSTEM

The proposed decision production approach was used in the previous chapter to derive an analytical formulation of a fixed reorder cycle inventory control system. However, the results obtained by the analytical method, based on a few assumptions, can be used to evaluate a limited set of information systems. In particular, the analytical solution is applicable to those situations where information available is current and relatively accurate. Therefore the next step in this research is to drop the assumptions of the analytical method, and extend the analytical solution to situations where information available is neither current nor very accurate. A simulation model will be used in this chapter to attain this objective.

The design of a simulation model requires careful planning. Many design decisions must be taken before a simulation model can be used to study a system. First, the experimenter must select an appropriate method for output data analysis. A final decision on the method of data analysis, however, can not be taken without examining the time required for the simulation to reach its steady state. Next, the reliability of output data must be ensured by selecting an appropriate length and number of replications for the simulation run. Finally, a suitable variance reduction technique should be used to improve the reliability of output data. Most of these decisions are based on data collected from

pilot runs of the simulation model. The first three sections of this chapter describe the details of the simulation design.

Once the simulation model is ready, it can be used to study the fixed reorder cycle system. However, an appropriate experimental design should be used to collect data from the simulation model. An advantage of the simulation method is that the simulation can be used to make some initial observations, and a suitable experimental design can be selected based on these initial observations. This two stage method is used in this chapter to select an appropriate experimental design for this research.

The data obtained from the simulation experiment is finally used to estimate the decision generation and gross payoff functions. The last few sections of this chapter are devoted to the estimation and interpretation of these functions. The effect of each input on output and the substitution possibility between each pair of inputs are also discussed for each of these functions.

As in Chapter 4, the fixed reorder cycle inventory system is examined here for two cases: (1) Lost sales case, and (2) Backordering case. Once more, the lost sales case is used as the primary focus of discussion, and the backordering case is handled by making appropriate modifications to the lost sales case. However, as stated before, the simulation design is discussed first in this chapter, starting with the methods for output data analysis in Section 5.1.

5.1 Output Data Analysis

The first step in designing a simulation is to examine the use of the simulation output data. The purpose of this simulation is to

investigate the behavior of the gross payoff and decision generation functions [(4.1) and (4.4)] of a fixed reorder cycle inventory system. In order to estimate these functions, various system configurations, each one corresponding to a specific combination of input levels, of the fixed reorder cycle system should be simulated. For each system configuration, observations must be obtained to estimate the steady-state expected value and confidence interval for the output variable.

Most statistical procedures designed for simulation output data analysis require collecting independent and identically distributed (i.i.d.) observations. There are three methods available for steady-state simulations aimed at producing observations that can be safely regarded as being i.i.d. Two of these methods, Batch means and Regenerative methods, can be based on a single "long" run of the simulation. The batch means method divides the simulation run into a number of batches of equal length, and uses the batch means for statistical inference procedures. The regenerative method, on the other hand, attempts to identify random times at which the process probabilistically "starts over", that is, regenerates, and uses these regeneration points to obtain independent observed values of random variables to which classical statistical analysis can be applied to form point and interval estimates. Both methods pose certain implementation problems. The principal difficulty of the batch means method is to select a batch size large enough to remove any harmful correlation between the batch means [Law 1977, Schmeiser 1982], whereas the problem of using the regenerative method in practice is that real world simulations may not have regeneration points or (even if they do) the

expected cycle length may be so large that only a few cycles can be simulated [Law and Kelton 1984, Ch. 8].

The third method for steady-state analysis makes several "shorter" independent replications. However, one difficulty with this method is that iid observations from the replications may not provide an unbiased estimate of the expected value of the output random variable because initial conditions may not be representative of the steady-state behavior of the system. In practice, this problem is overcome by discarding some observations in the process of bringing the model into steady-state conditions.

The decision to use the method of replication should be made against the increase in the relative cost of obtaining each retained observation [Schriber and Andrews 1981]. If the time taken to attain steady-state is small compared to the run length, the choice of the method of replication is justified by its ability to provide iid observations. On the other hand, if the simulation takes a long time to reach steady-state, either the batch means or the regenerative method should be used. Thus a final decision on the method of data analysis can not be made before examining the length of the transient state behavior. For this purpose, a few pilot runs are used in the next section to estimate the time required to attain steady-state conditions.

5.2 Steady-State Conditions

A system is said to be in its steady-state if the probability of ~~being in one of its states is governed by a fixed probability function;~~ otherwise it is in the transient state [Kleijnen 1975, p. 69]. Several procedures are available in the simulation literature for the detection

of steady-state conditions [see, for example, Wilson and Pritsker 1978]. However, the statistical efficacy of any such procedure must be examined by comparing the bias reduction against the associated increase in variance due to a reduction in the simulated time period [Turnquist and Sussman 1977]. A survey by Gafarian et al. [1978] indicates that many of the traditional procedures for identifying steady-state conditions exhibit very poor performance. Most of these methods overestimate the length of the transient state, and thus increase simulation cost considerably.

Two relatively new procedures suggested by Kelton and Law [1983] and Schruben [1982] are based on sound theoretical considerations, and overcome the problems associated with the earlier procedures. In this section, both these procedures are used together to identify steady state conditions for this simulation. First, the data collection for these two methods is described, followed by a discussion of the choice of initial conditions. A visual inspection of the transient state behavior is performed next to get an understanding of the extent of initial bias. Finally, each of the two methods are used separately for the detection of steady state conditions. The method of data collection is described next.

Data Collection: The same set of data can be used for both Kelton and Law [1983] and Schruben [1982] procedures. Four pilot runs are used for this purpose to collect the required data to identify steady state conditions. These pilot runs correspond to four different levels of the dependent variable (gross payoff): High (Run #1), Medium high (Run #2), Medium low (Run #3), and Low (Run #4).

For each pilot run, the average of five independent replications are

used to obtain a time series of the dependent variable. Next the replication averages are grouped into "batches" to form n batch means, each being the mean of five adjacent values of the average time series. The n batch means then constitute the final data points for the subsequent statistical analysis.

Initial Conditions: The simulation can be started with any initial conditions insofar as any initial conditions will lead to the same steady state. However, it is still relevant to select "good" starting conditions to reduce the magnitude of initial bias [Kleijnen 1975, p. 70]. For the simulated fixed reorder cycle system, gross payoff for the first five days is fully determined by initial conditions prevailing on the first day. The decision taken at the end of the first day takes effect only on the sixth day because the quantity ordered, if any, on the first day arrives at the beginning of the sixth day. However, the gross payoff on sixth day onward is not affected by initial conditions if there is no carry over inventory on the sixth day. The condition of no carry over inventory on the sixth day occurs if the opening quantity on hand on the first day is set equal to the total demand of the first five days, and the opening quantity on order is set to zero. Thus the initial conditions - quantity on hand and quantity on order - are set accordingly, and observations are taken from sixth day onwards.

Visual Inspection: Before undertaking the statistical analysis, a visual inspection of the output is performed to determine the extent of initial bias. As suggested by Schriber [1974, p. 121], the output variable (gross payoff) is observed at fixed intervals. The interval used here corresponds to twenty five decision periods. The level of gross payoff during the interval and the accumulated value are given for

one pilot run in Table 5.1. A visual examination of the data in this table and similar data for three other pilot runs does not reveal significant initial bias. This preliminary result indicates that the length of the transient period in this simulation may not be very long. This hypothesis is tested below by Kelton and Law method and Schruben method for initial bias detection.

Kelton and Law Method: According to this method, a simulation attains steady-state conditions when the slope of the output time series becomes zero. The output data (Y_t ; $t=1, \dots, n$) is used to fit the following model:

$$(5.1) Y_t = u + at + e_t$$

where $E(Y_t) = u$ and $E(e_t) = 0$. Since the output time series is correlated, an ordinary least squares (OLS) regression cannot be used estimate equation (5.1). To overcome this problem, the authors recommend the use of Amemiya's [1973] Generalized Least Squares (GLS) procedure. The null hypothesis that the slope of the output time series (a) is equal to zero can be tested using the Amemiya's method.

Table 5.2 gives the significance level of the zero-slope hypothesis for four pilot runs. A high value of the significance level indicates strong support for the zero-slope hypothesis. For each pilot run, the value of the significance level is given for five cases. Each case is based on 65 data points. However, the starting point for each case is different: Case 1: 1, Case 2: 16, Case 3: 31, Case 4: 46, and Case 5: 61. With no deletion (Case 1), a significant initial bias is seen in one out of four runs (Run 3). This initial bias, however, becomes insignificant if 15 data points are deleted (Case 2). The results of this method are next examined in light of the Schruben procedure.

TABLE 5.1 : TRANSIENT STATE BEHAVIOR OF THE SIMULATION MODEL

Time Interval	Interval Value	Cumulative Value
1	94.21	94.21
2	93.33	93.77
3	91.69	93.11
4	92.34	92.91
5	93.27	92.98
6	91.40	92.70
7	92.76	92.70
8	93.05	92.74
9	92.53	92.70
10	94.24	92.85
11	91.35	92.73
12	93.36	92.78
13	92.08	92.72
14	92.91	92.73
15	90.83	92.60
16	93.09	92.63
17	92.79	92.64
18	94.07	92.72
19	91.79	92.67
20	92.55	92.66
21	92.96	92.68
22	92.01	92.65
23	93.34	92.68
24	88.80	92.51
25	92.28	92.50

TABLE 5.2: LEVEL OF SIGNIFICANCE FOR INITIAL BIAS

RUN	CASE:1	CASE:2	CASE:3	CASE:4	CASE:5
1	.9363	.4009	.4953	.7136	.5580
2	.4771	.7456	.3362	.3070	.2202
3	.0373	.2341	.4876	.2142	.8597
4	.1125	.2688	.8886	.3486	.9883

Schruben Method: This method prescribes a test for stationarity in the output mean based on the asymptotic convergence of partial sums of deviations about the average of a Brownian bridge process. This test is applicable for a specific sign of the initial bias. In this case this test is applied for both positive and negative bias. The results indicate, if at all, the presence of a weak positive bias.

The data generated for the Kelton method can also be used to test initial bias using the Schruben procedure. This procedure returns the value of a F-stat. A F-stat greater than a preassigned $F(3,3)$ value indicates presence of bias. The values of $F(3,3)$ for $\alpha = .01, .05,$ and 0.1 are 29.5, 9.28, and 5.39 respectively. Table 5.3 gives six values of the F-stat for each pilot run. The range of data points used in each of the six cases is as follows: Case 1: 1-50, Case 2: 6-55, Case 3: 11-60, Case 4: 16-65, Case 5: 21-70, and Case 6: 26-75.

The results of this analysis confirms the absence of strong initialization bias. With no deletion (Case 1), the initial bias is significant for one pilot run (Run 4) for an alpha of 0.1. This bias reduces rapidly if 5 or 10 data points are deleted.

Kelton-Law procedure was repeated with 5 and 10 initial data points deleted to check if it supports the results obtained using the Schruben

test. This additional test data (Table 5.4) corroborate that the initial bias can be disregarded if 5 or 10 data points are deleted. In conclusion, a conservative estimate for the transient period will be the first 10 data points. Since each data point represents 5 decision periods, the deletion point is fixed at 50 decision periods.

TABLE 5.3: F-STAT FROM SCHRUBEN METHOD

RUN	CASE:1	CASE:2	CASE:3	CASE:4	CASE:5	CASE:6
1	3.878	1.239	0.653	0.053	1.680	1.794
2	0.010	0.655	0.065	0.099	0.063	0.134
3	4.544	3.694	2.996	2.080	1.160	0.646
4	6.343	3.046	1.526	0.693	0.487	0.836

Since the time taken for this simulation to reach steady-state is reasonable, the method of replications is the best choice for output data analysis (see section 5.1). However, a few additional design decisions have to be taken before this simulation can be used to study a fixed reorder cycle system. All of these decisions refer to the reliability of output data, and are discussed in the following section.

TABLE 5.4:

LEVEL OF SIGNIFICANCE FOR INITIAL BIAS

RUN	NO DELETION	DELETION=5	DELETION=10
1	.9363	.1302	.5833
2	.4771	.2601	.8723
3	.0373	.9296	.4178
4	.1125	.4259	.6489

5.3 Reliability of Output Data

The overall objective of the design decisions taken in this section is to ensure the reliability of simulation output data. The decisions taken in this section includes the following: Variance Reduction method, Number of Replications, and Simulation Run Length. Each of these decisions has a bearing on the variance of the output estimate, and thus affects the reliability of output data.

Variance Reduction: Various techniques have been proposed to reduce the variance of simulation output. Two of these methods, Common Random Numbers and Antithetic Variates, are easy to implement and have been widely used in simulation experiments [Law and Kelton 1984, Ch. 11]. The common random numbers method uses the same series of pseudo-random numbers for multiple system configurations to allow comparison of alternative systems under identical conditions. The use of common random numbers, however, induces correlation across output data points, and makes statistical data analysis (e.g., regression) difficult. Thus the common random numbers method is ruled out for this simulation.

The method of antithetic variates, on the other hand, makes pairs of system runs using complementary random numbers to attempt to induce negative correlation between two observations in the pair. Thus if the average of the two observations in the pair is used as a basic data point in the analysis, this average should produce iid observations with lower variance. However, additional programming has to be undertaken to ensure that the random number sequences used for the two parallel runs within a pair are synchronized throughout the run length.

Since the success of antithetic variates is model dependent, pilot tests should be performed to check its efficacy in the present context.

Four pilot settings used in Section 5.2 are employed to compare the variance obtained by using 20 independent replications (Case 1) against 10 pairs of antithetic runs (Case 2). For both cases, each run consists of 350 decision periods of which 50 initial points are disregarded owing to initialization bias. Table 5.5 shows that the mean values of the dependent variable, gross payoff, obtained by the two methods are about the same. However, on the average, the antithetic variates reduce the output standard deviation by about 50%. Moreover, it is found that the CPU time required by a pair of antithetic runs is significantly less than that required by a pair of independently seeded replications. Thus it is concluded that antithetic variates should be used for the simulation of the fixed reorder cycle system.

TABLE 5.5: VARIANCE REDUCTION USING ANTITHETIC VARIATES

RUN	CASE : 1		CASE : 2		% REDN.
	MEAN	SD DEV	MEAN	SD DEV	
1	92.30	0.73	92.41	0.43	41.10
2	74.55	1.14	74.98	0.53	53.51
3	40.08	2.69	39.91	1.23	54.28
4	16.30	3.51	15.96	1.85	47.29

Run Length and Number of Replications: Since the method of replications, when properly conducted, provides iid observations, the mean of the replication means can be used as an estimate for the expected value of the output variable. In order to ensure a uniform level of reliability across data points, it is important that the variance of the output data points lie within a reasonable range. However, it is unlikely that a single choice of run length and number of

replications would achieve this objective. Consider, for example, the data presented in Table 5.5. Although all four data points are obtained under identical conditions (equal run length and number of replications), they lead to different levels of variance. Thus, the remaining two design variables, simulation run length and number of replications, should not be based on a single set of values for all data points.

A stepwise strategy can be used to select the appropriate values of the two design variables. This strategy is based on a trend found in the data in Table 5.5 (and elsewhere) that the output variance tends to increase with a decrease in the level of gross payoff. The output data points are collected in a sequential manner starting with high values of inputs (hence high levels of output) keeping both run length and number of replications at some fixed values. As soon as the output variance exceeds a threshold level, both run length and number of replications are increased to lower the variance level. This process is repeated a few times until all data points are collected.

Finally, certain additional considerations must be taken into account before selecting specific values for run length and number of replications. First note that the output variance reduces if the run length (say m observations) or number of replications (n) is increased. But for a total number of observations of N (based on a given computing budget), $N = mn$. Thus an increase in run length has to be traded against a reduction in the number of replications, and vice versa.

The results given in a recent study by Kelton and Law [1984] are used to examine the trade off between run length and number of replications. For example, to start with, the total number of

observations for this simulation is fixed in the neighborhood of $N=3000$ so as not to exceed the given computing budget. With this constraint in mind, a suitable number of replications should be chosen to control the level of output variance. For $N=2880$, the Kelton and Law study found that increasing the number of replications to more than 10 may in effect adversely affect the statistical properties of the mean estimate. For this simulation, a comparison of three different number of replications ($n = 5, 10, 20$) is found inconclusive. Thus based on the Kelton and Law study, the number of replications is fixed at $n=10$.

With the simulation design ready, the simulation model can be used to evaluate a fixed reorder cycle system. In the next section, an appropriate experimental design is selected to collect data to estimate the decision generation and gross payoff functions for this system.

5.4 Experimental Design

The design of the simulation experiment should be considered in light of the objectives of this study. Thus the objectives of this experiment are reviewed first, and an appropriate design is selected based on this review.

The simulated fixed reorder cycle system is used to investigate the following questions:

1. What is the effect of increasing information accuracy and coverage on decision accuracy? What are the possibilities of substitution between each pair of inputs (e.g., quantity on record accuracy and order-up-to point accuracy)?
2. What are the impacts of increasing decision accuracy and coverage on gross payoff? Are the marginal products positive but diminishing?

What are the possibilities of substitution between decision accuracy and coverage?

The answers to these questions require the knowledge of both decision generation (4.4) and gross payoff functions (4.1). However, there is a wide range of possibilities for the exact functional forms for the two production functions [Mukhopadhyay 1986]. Thus a choice of specific functional forms without prior knowledge of the properties of the production functions may lead to incorrect results. To overcome this problem, the simulation experiment is designed in two parts. In the first part of this experiment, preliminary simulation runs are made to study the individual effects of each input on the output variable. The results of the first part can then be used to select an appropriate design for the second part of this experiment to examine the joint effects of all inputs on the output variable. The two parts of this simulation experiment are described in the remainder of this section.

Part I: In this part, the fixed reorder cycle simulation is run to study the effect of each input variable (of both decision generation and gross payoff functions) on the output variable while keeping all other input variables at constant levels. Based on this data, the following properties of the production functions can be examined:

1. Marginal Product of Inputs: The total product¹ curve of each input is plotted for two different values of the other input(s) for both the functions. The plots thus obtained [Figures 5.1 to 5.6; see Appendix A3.2 also] indicate that as the amount of an input is increased, the value of the total product also increases although at a decreasing rate.

¹The total product of an input is defined as the quantity of output produced from the input if all other inputs are kept at constant levels.

FIGURE 5.1: TOTAL PRODUCT OF DECISION ACCURACY

Curve 1: Decision coverage = 0.25

Curve 2: Decision coverage = 0.20

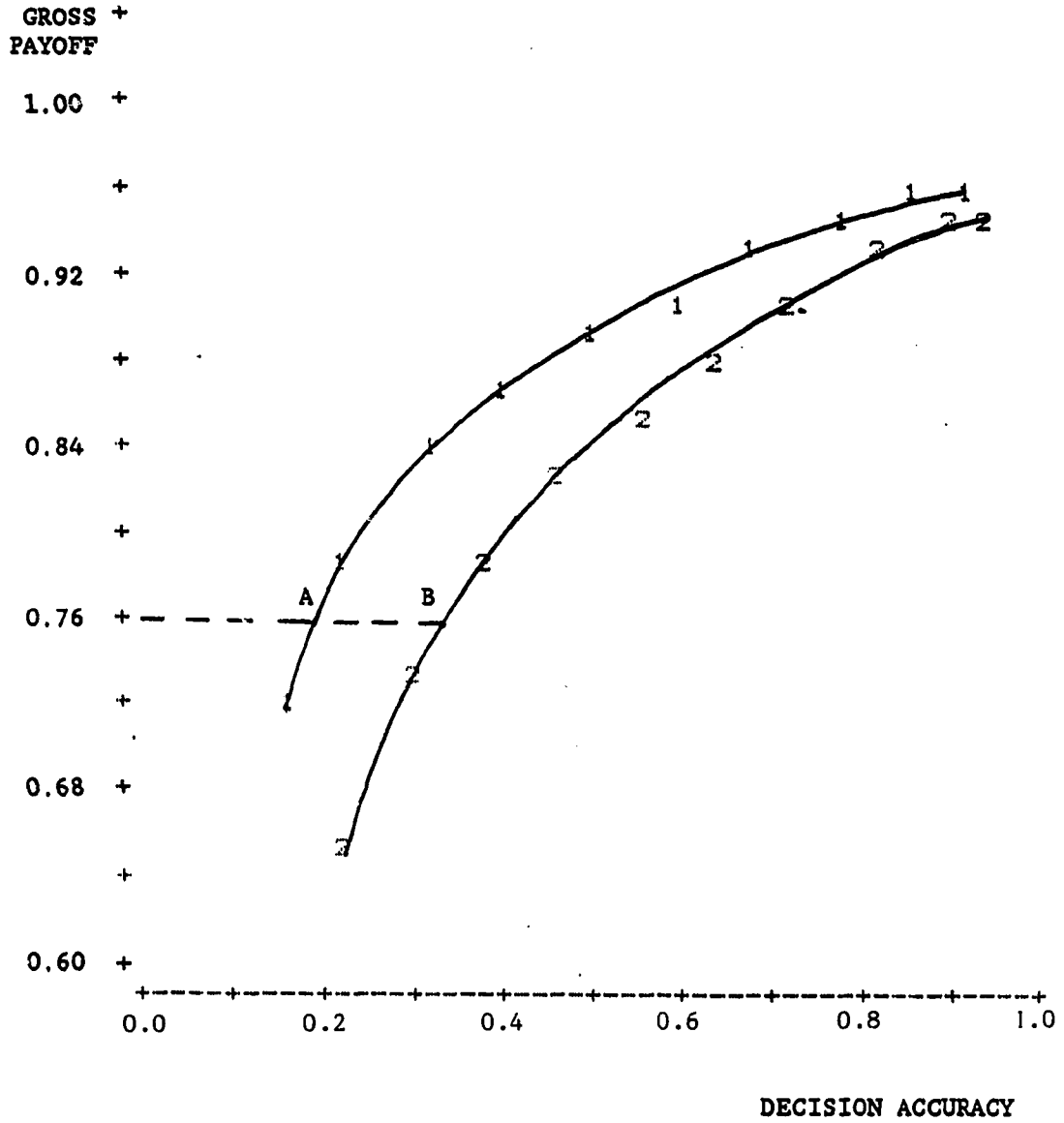
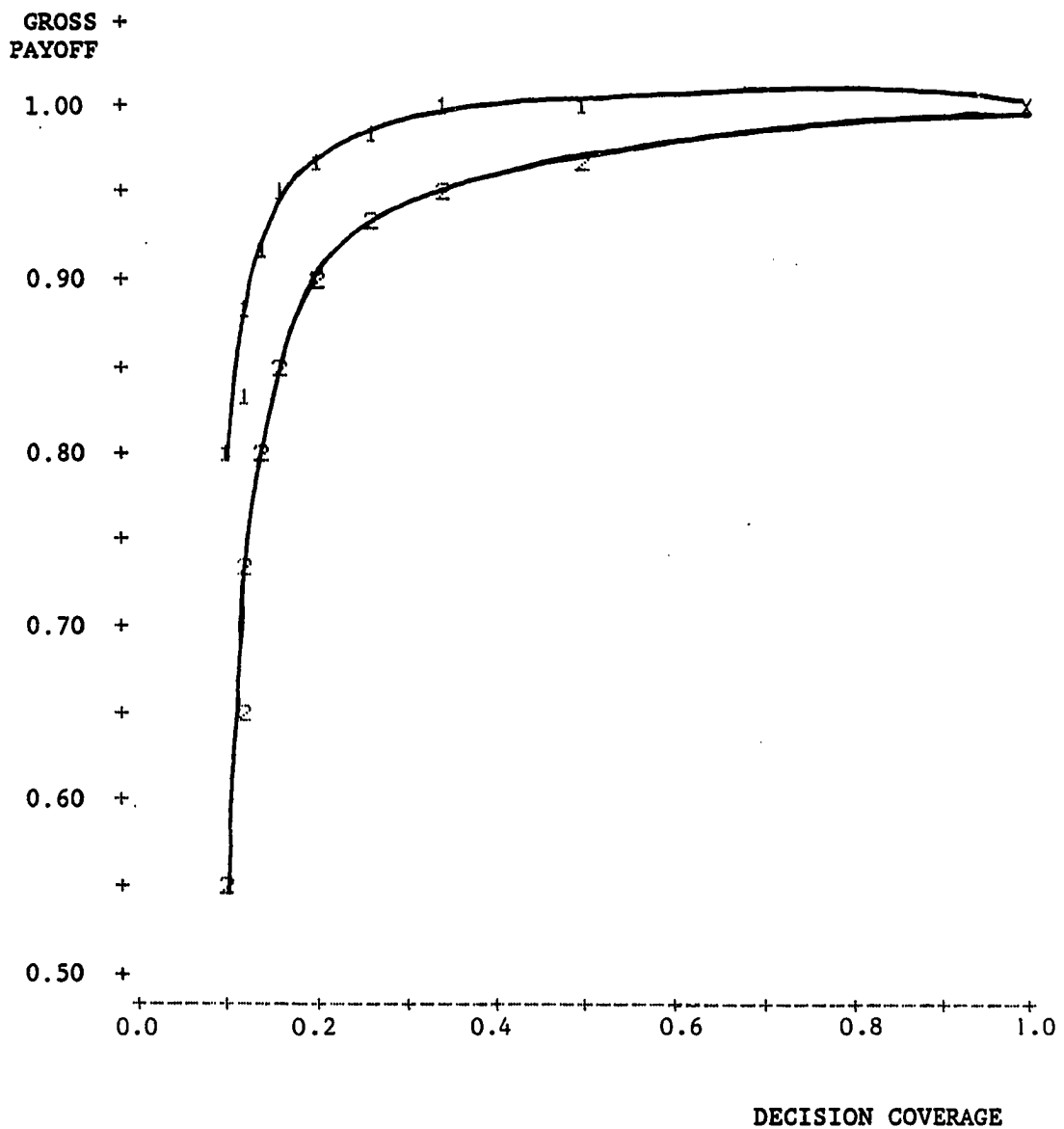


FIGURE 5.2: TOTAL PRODUCT OF DECISION COVERAGE

Curve 1: Decision accuracy = 1.00

Curve 2: Decision accuracy = 0.70



DECISION COVERAGE

FIGURE 5.3: TOTAL PRODUCT OF ORDER-UP-TO POINT ACCURACY

Curve 1: Quantity on record accuracy = 0.55
 Quantity on record coverage = 0.25
 Order-up-to point coverage = 0.40

Curve 2: Quantity on record accuracy = 0.70
 Quantity on record coverage = 0.20
 Order-up-to point coverage = 0.30

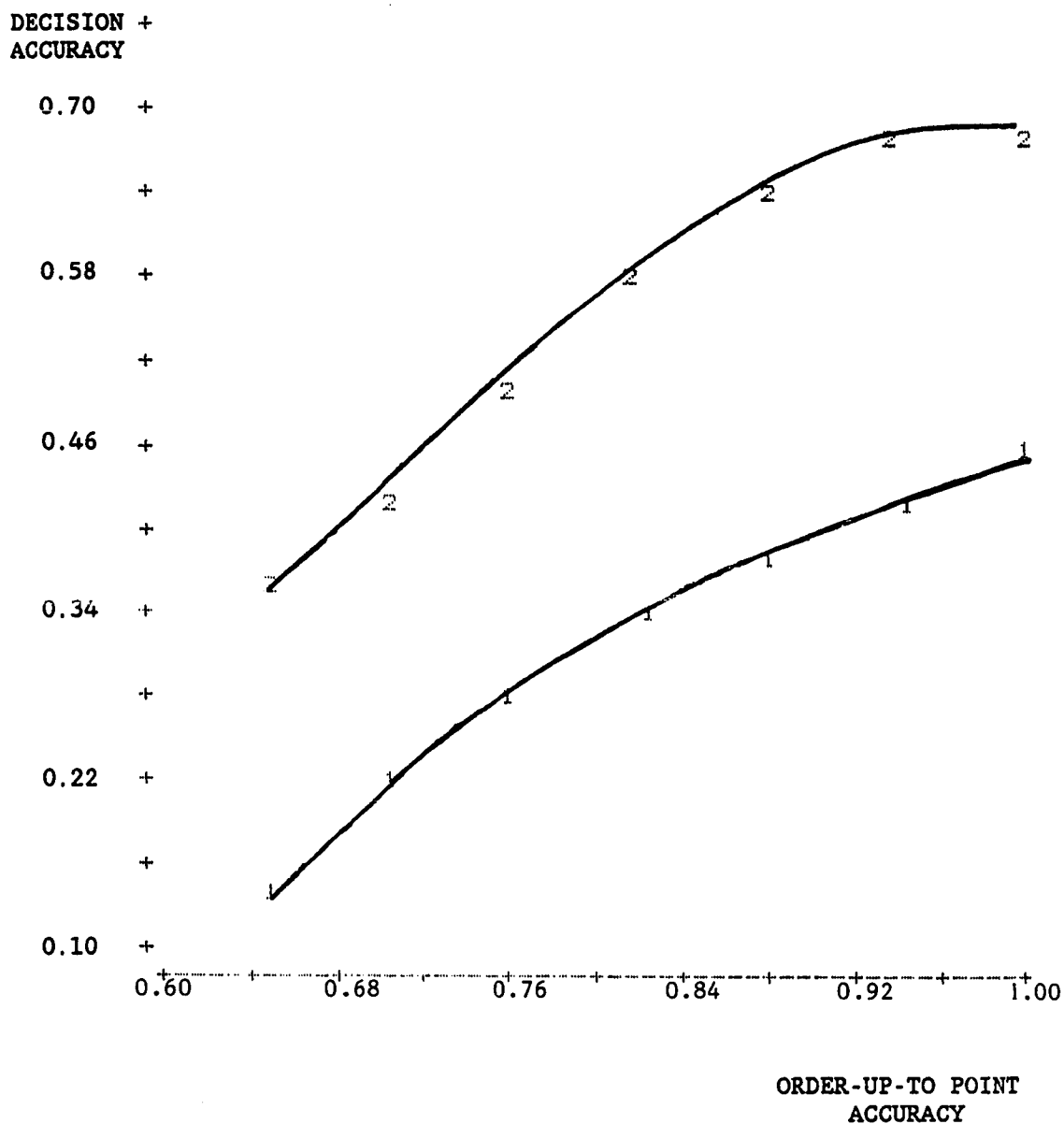


FIGURE 5.4: TOTAL PRODUCT OF QUANTITY ON RECORD ACCURACY

Curve 1: Order-up-to point accuracy = 0.78
 Quantity on record coverage = 0.25
 Order-up-to point coverage = 0.50

Curve 2: Order-up-to point accuracy = 0.93
 Quantity on record coverage = 1.00
 Order-up-to point coverage = 0.24

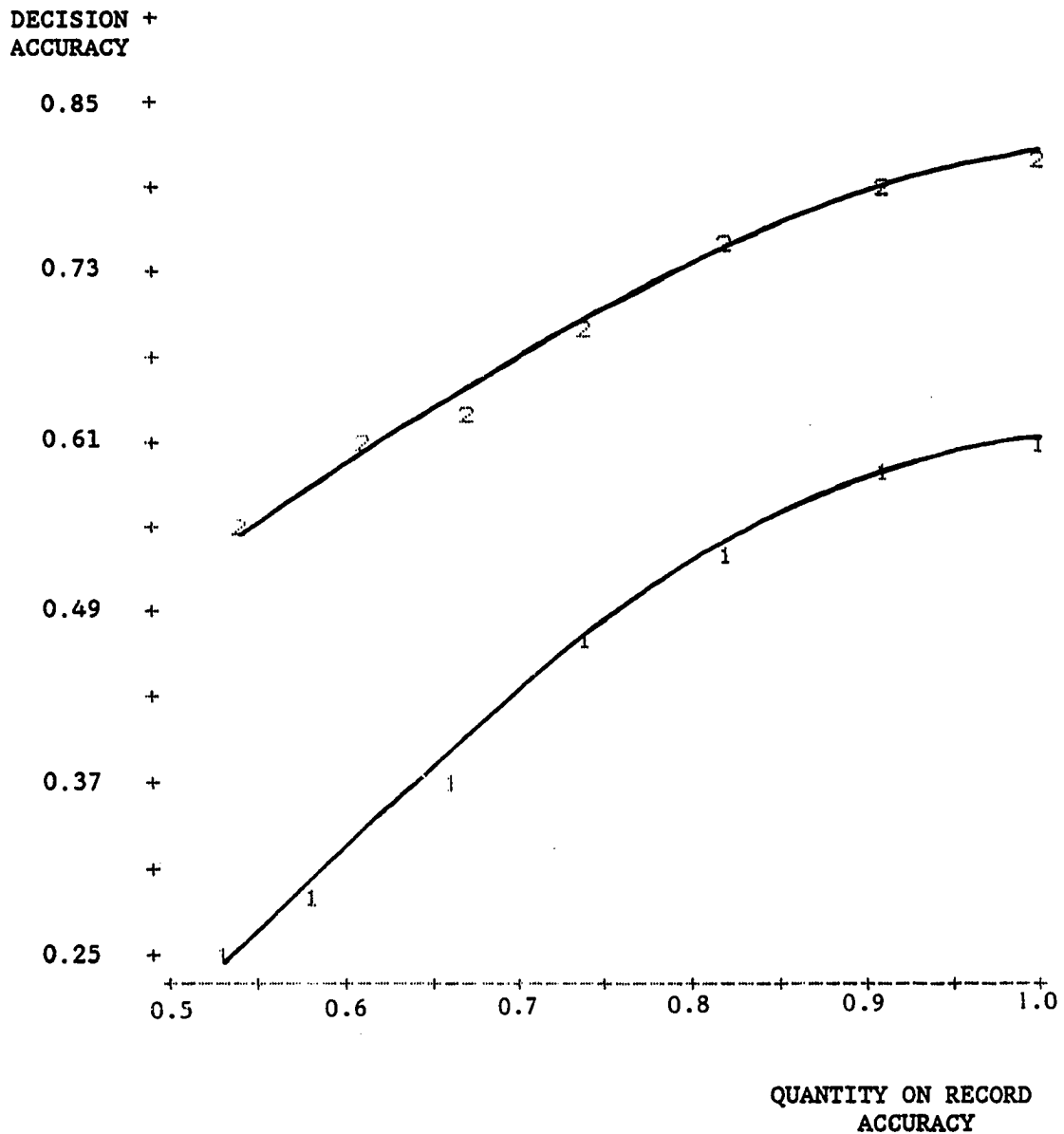


FIGURE 5.5: TOTAL PRODUCT OF ORDER-UP-TO POINT COVERAGE

Curve 1: Quantity on record accuracy = 0.60
 Quantity on record coverage = 0.25
 Order-up-to point accuracy = 0.70

Curve 2: Quantity on record accuracy = 0.80
 Quantity on record coverage = 0.50
 Order-up-to point accuracy = 0.80

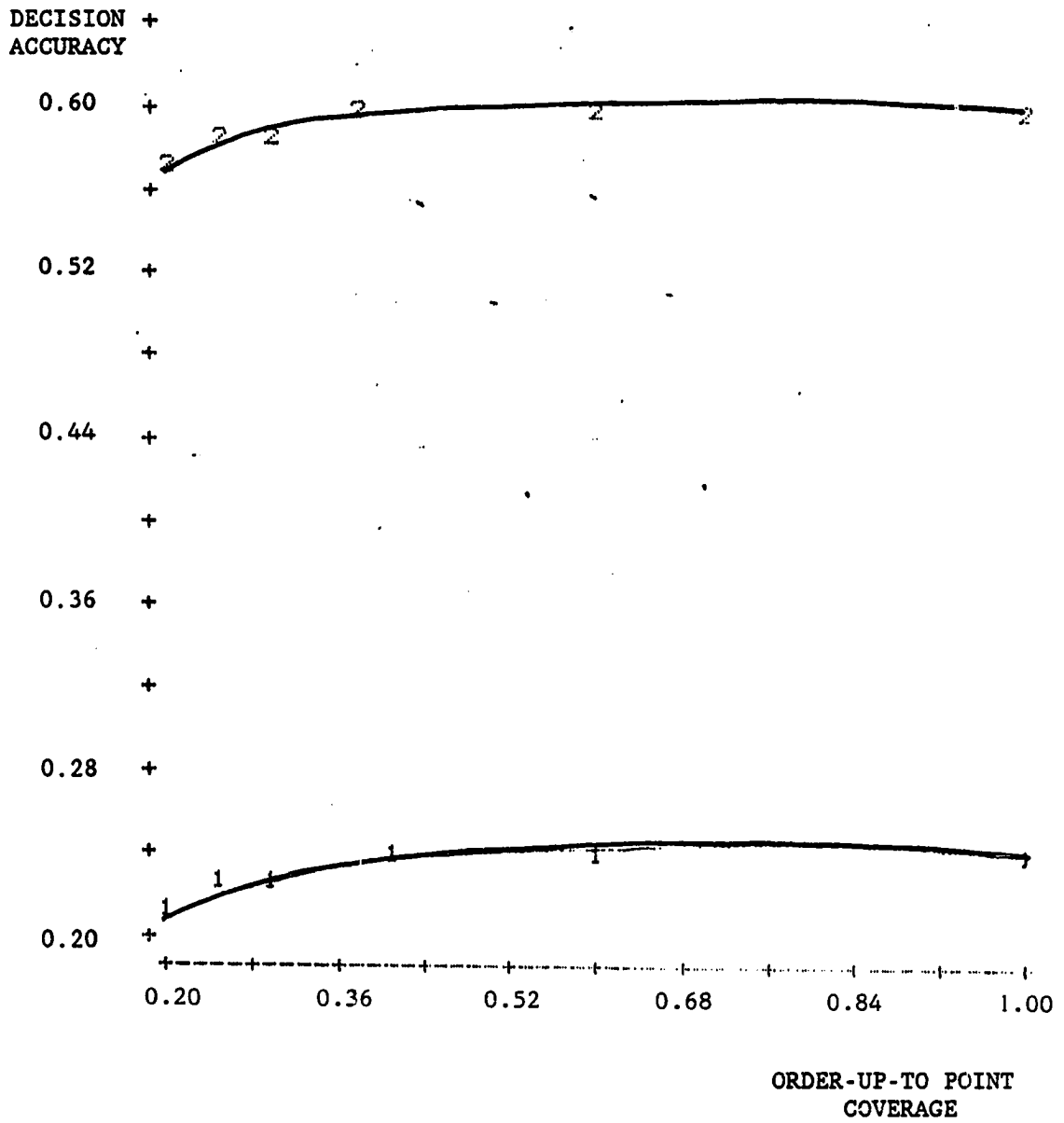
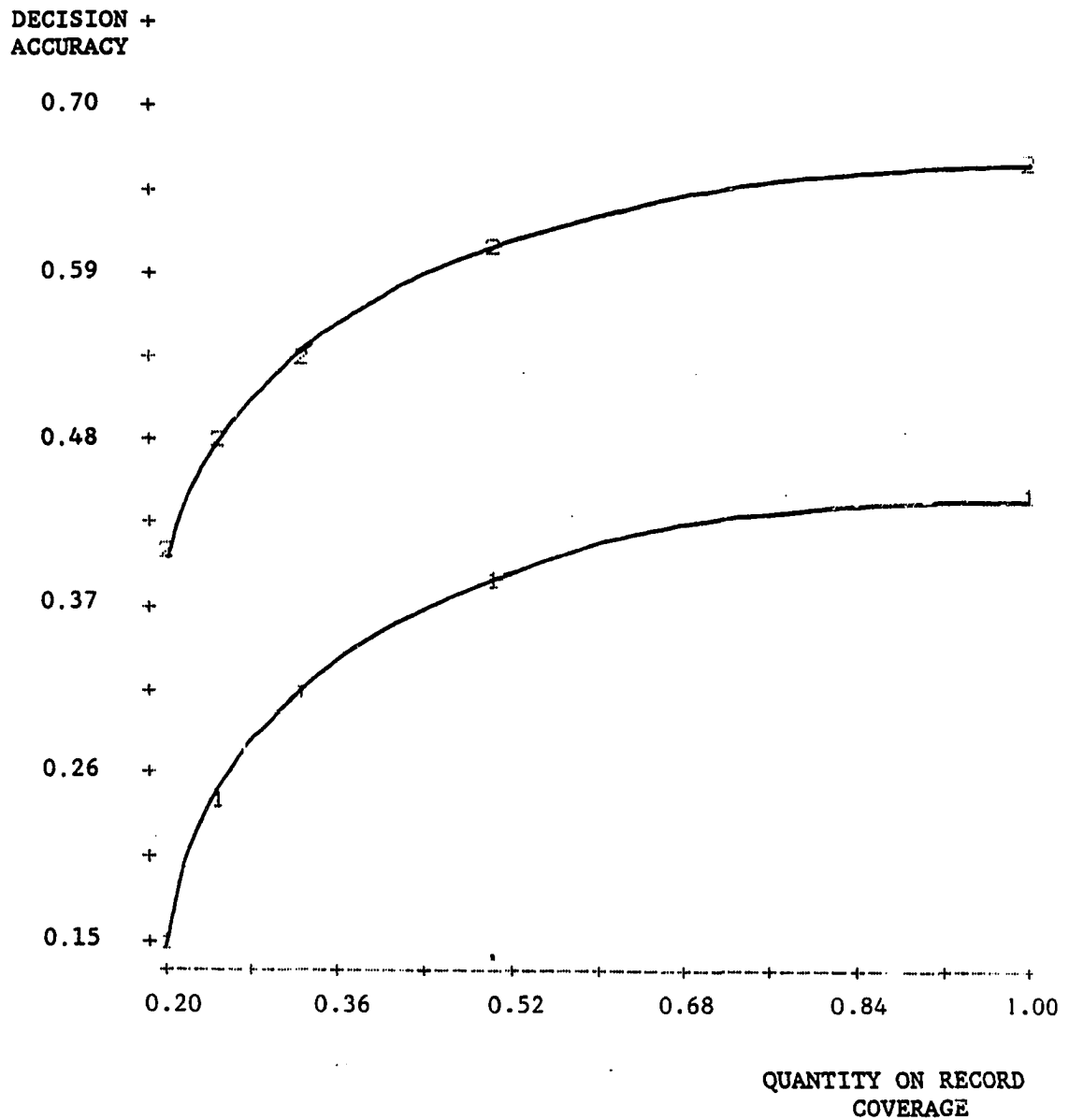


FIGURE 5.6: TOTAL PRODUCT OF QUANTITY ON RECORD COVERAGE

Curve 1: Quantity on record accuracy = 0.485
 Order-up-to point accuracy = 0.78
 Order-up-to point coverage = 0.40

Curve 2: Quantity on record accuracy = 0.605
 Order-up-to point accuracy = 0.90
 Order-up-to point coverage = 1.00



In other words, the marginal product of each input of both gross payoff and decision generation functions is positive but diminishing. For example, increasing decision accuracy, keeping decision coverage at a constant level, increases gross payoff although at a decreasing rate [Figure 5.1]. In short, the shape of the total product curves supports the neoclassical assumption that the marginal product of an input is positive but diminishing in the relevant range of production.

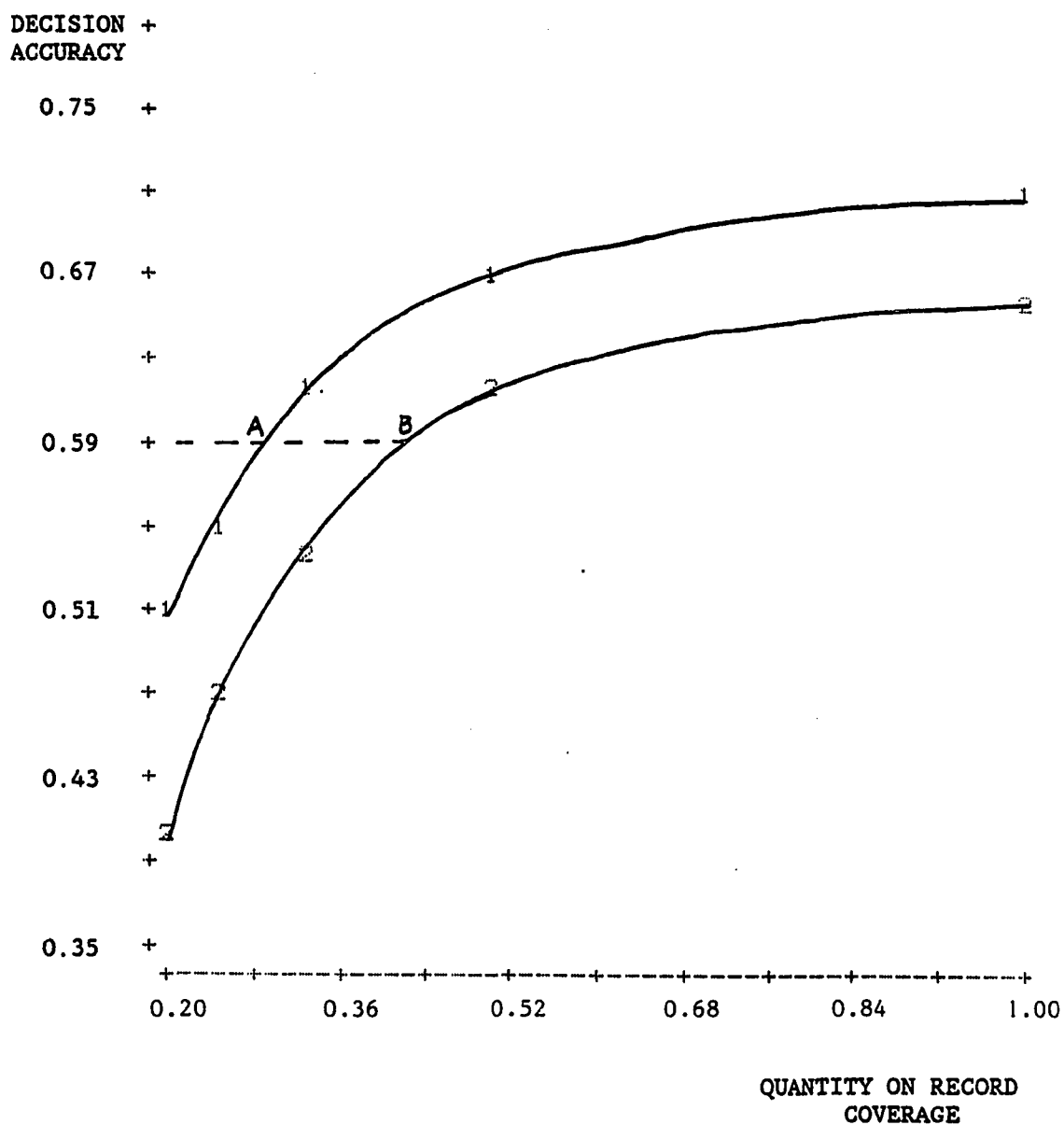
2. Negative Tradeoffs between Input Pairs: The preliminary data obtained in this phase can be used to check negative tradeoffs between input pairs. First consider the possibility of substitution between decision accuracy and coverage in the gross payoff function. For a given level of gross payoff, two equivalent input combinations can be obtained from the total product curve of Figure 5.1 or 5.2. These input combinations can be used to check if there is a negative tradeoff between input pairs. For example, consider the two input combinations A and B in Figure 5.1. Input combination A requires a lower level of decision accuracy and a higher level of decision coverage than input combination B to generate the same amount of gross payoff. In other words, there is a negative tradeoff between the two inputs of the gross payoff function.

Since the decision generation function (4.4) consists of four inputs, it results in six possible input pairs. To check the substitution possibilities between each input pair, total product curves of one of the inputs in the pair must be obtained for two values of the other input keeping all other inputs at constant levels. For the sake of brevity, only one such input pair, quantity on record accuracy and coverage, is illustrated in Figure 5.7.

FIGURE 5.7: NEGATIVE TRADE-OFF BETWEEN
QUANTITY ON RECORD ACCURACY AND COVERAGE

Curve 1: Quantity on record accuracy = 0.70
Order-up-to point accuracy = 0.90
Order-up-to point coverage = 1.00

Curve 2: Quantity on record accuracy = 0.60
Order-up-to point accuracy = 0.90
Order-up-to point coverage = 1.00



Alternatively, the negative tradeoff between any input pair can be examined algebraically. For example, consider the decision generation function (4.4) for given levels of order-up-to point accuracy and coverage (R'_a, R'_c):

$$(5.2) D_a = g(R'_a, R'_c, Q_a, Q_c).$$

The total differential of the production function (5.2) is given by:

$$(5.3) dD_a = g_a dQ_a + g_c dQ_c;$$

where g_a and g_c are partial derivatives of D_a with respect to Q_a and Q_c . Thus, for a given level of D_a (or $dD_a = 0$), the tradeoff between Q_a and Q_c can be examined by (5.4) below:

$$(5.4) dQ_a / dQ_c = - g_c / g_a.$$

Based on the total product curves in Figures 5.4 and 5.6, assume that both g_a and g_c are positive. Thus the right hand side of (5.4) is negative. In other words, there is a negative tradeoff between quantity on record accuracy and coverage. This procedure can be repeated to produce the same result for any pair of inputs for the decision generation as well as the gross payoff function.

Part II: Based on the results obtained above, the following production functions can be eliminated from further consideration:

1. Linear production functions require that the marginal product of each input is positive and constant at all output levels. Since marginal product of each input is found to be positive but diminishing, linear production functions should not be selected as the appropriate functional forms.

2. Fixed proportion production functions rule out substitution possibilities between any pair of inputs. However, the negative tradeoffs between input pairs evidenced in Part I indicate substitution

possibilities between each input pair. Hence fixed proportion production functions are also excluded from further consideration.

The elimination of the above functional forms limits the choice to the following production functions: Cobb-Douglas, Constant Elasticity of Substitution, Variable Elasticity of Substitution, Translog function, or any other suitable form. However, the data generated thus far do not allow one to accept or reject any of these functional forms. Hence the final choice of the parametric forms for the production functions must be left to the second part of the simulation experiment.

Since several possibilities exist for the final forms of the production functions, the data generated in Part II of this experiment should allow maximum flexibility in the estimation of the production functions. Since fractional factorial designs limit the possible set of mathematical models for the response variable [Kleijnen 1975, p. 320], full factorial designs are used to generate data for the estimation of the gross payoff and decision generation functions.

The final designs for the estimation of both gross payoff and decision generation functions should be considered using two opposing criteria: (1) Sufficient data points must be obtained to ensure high degree of reliability of the estimated functions, but (2) the total number of observations must be limited so as not to inflate the computing budget. The final experimental designs are given below for each of the production functions:

Decision Generation Function: The data obtained in Part I of this simulation can be used to screen the important variables for the decision generation function [Kleijnen 1975, p. 77]. A careful examination of the total product curves in Figures 5.3 - 5.6 reveals

that the effect of order-up-to point coverage on decision accuracy is minimal. This result can be further illustrated through pairwise comparisons of the total products of order-up-to point coverage and the three other variables of the decision generation function. Figure 5.8, for example, compares the effects of order-up-to point coverage and quantity on record coverage on decision accuracy. It is clear from Figure 5.8 that the effect of order-up-to point coverage on decision accuracy is very low compared to that of quantity on record coverage.

There is also an intuitive explanation for the relatively low impact of order-up-to point coverage on decision accuracy. First note that order-up-to point indicates the gross requirement of an item for the review period. Next recall from Section 4.4 that order-up-to point coverage is the frequency with which the order-up-to point information is updated. Thus the relatively low effect of order-up-to point coverage on decision accuracy indicates that very little can be gained by updating the order-up-to point information on a frequent basis.

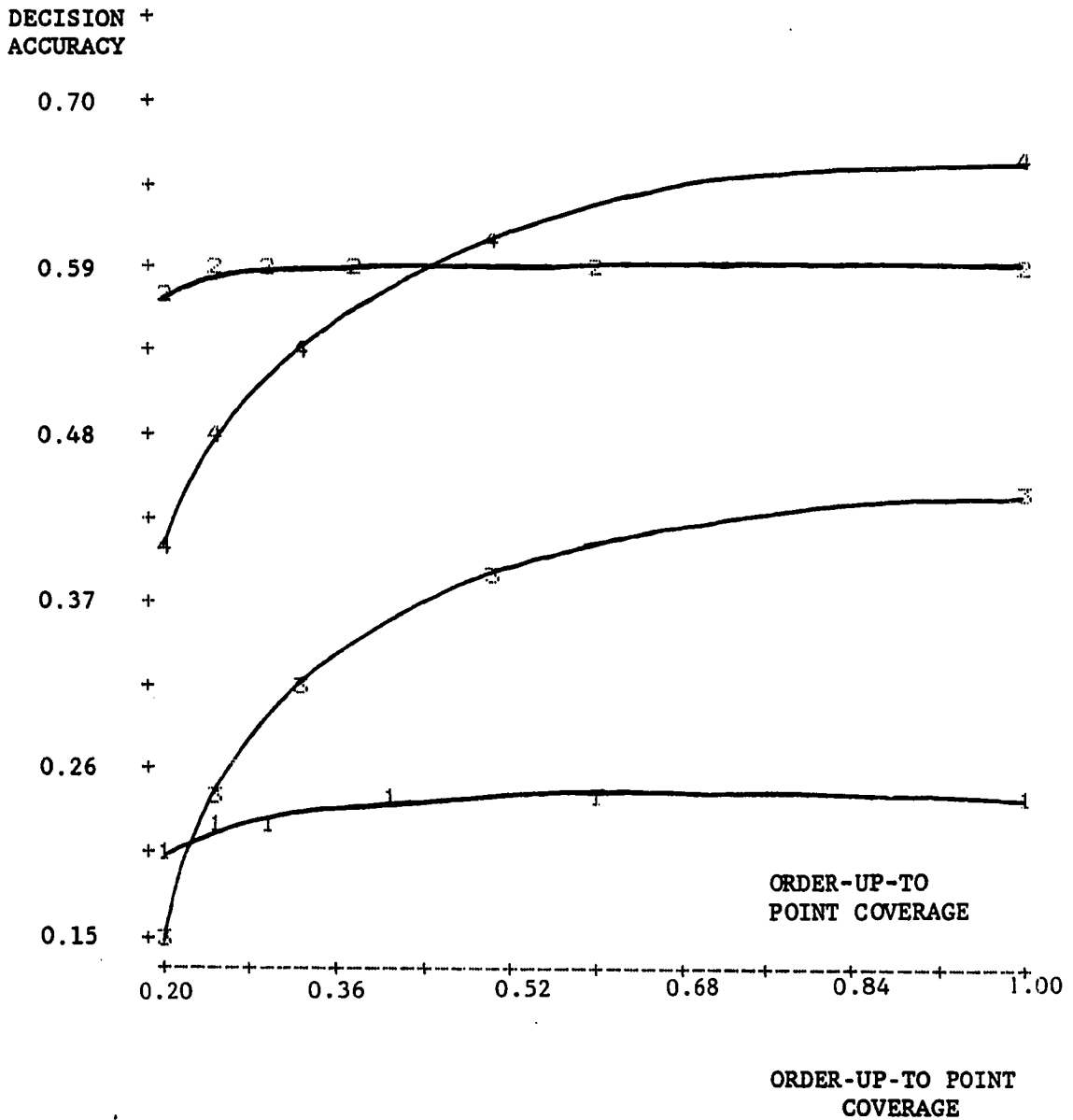
This interpretation is reasonable considering the stationarity of the demand distribution. Since the demand distribution is stable over the production period [see Section 4.5], it is reasonable that increasing the updating frequency for the order-up-to point information has little impact on decision accuracy. From this viewpoint, the low impact of order-up-to point coverage is considered an indirect validation of the simulation model.

Considering both the evidence from the data generated in Part I and the intuitive explanation given above, order-up-to point coverage is eliminated from the final design. Thus the reduced decision generation function to be estimated can be given as:

FIGURE 5.8: A COMPARISON OF THE EFFECTS OF ORDER-UP-TO POINT AND QUANTITY ON RECORD COVERAGE

Curve 1 and 2: Total product of order-up-to point coverage

Curve 3 and 4: Total product of quantity on record coverage



$$(5.5) D_a = g(R_a, Q_a, Q_c).$$

The experimental design used for this model estimation is a 5 X 5 X 5 full factorial design with 125 data points.

Gross Payoff Function: Both the factors of this function have substantial impacts on the dependent variable [Figures 5.1 and 5.2]. Therefore a 10 X 10 full factorial design with 100 data points is adopted for this function estimation.

The data obtained in Part II are used to estimate both the gross payoff and decision generation functions. The estimation procedure and the estimated functions are discussed in the following section.

5.5 Estimation of Production Functions

A two step procedure is followed for the estimation of both gross payoff and decision generation functions. First, a set of candidate functional forms is identified. Second, the functional form that explains the highest proportion of the variance in the response variable is selected as an approximation for the "true" production function. The estimation procedure and its results are discussed for both functions in the remainder of this section.

Gross Payoff Function: The data obtained in the first part of the simulation experiment indicated strong support for the elimination of linear production functions and fixed proportion production functions from the set of candidate functional forms. As a result, the choice of the final functional form is limited to the following functions: Cobb Douglas [CD], Constant Elasticity of Substitution [CES], Sato-Revankar Variable Elasticity of Substitution [VES], and Translog function. Before estimating these candidate production functions, a brief

comparison of the four functional forms and their estimation procedures are discussed below.

An important difference between the four candidate functions lies in their treatment of the elasticity of substitution which is the proportionate rate of change of the input ratio divided by the proportionate rate of change of the rate of technical substitution. The elasticity of substitution is a local measure of substitution possibilities between two inputs, and is a nonnegative number independent of the units of inputs and outputs. The magnitude of the elasticity of substitution indicates the ease with which one input can be substituted by another. The Cobb Douglas function assumes a unitary elasticity of substitution, whereas the CES function allows a constant value for the elasticity of substitution on the positive real axis. The family of VES functions, on the other hand, allows the elasticity of substitution to change depending on the input combinations. Finally, the translog function also allows nonunitary elasticity of substitution.

A second way of comparing the four functions is to consider the Cobb Douglas function as the basic form and the three other forms as attempts to generalize the basic form. In fact, it can be shown that under certain conditions, each of the three other functions reduces to the Cobb Douglas form. Of the three generalization of Cobb Douglas, the translog function is the most flexible form since it provides a second order approximation to an arbitrary production function. However, the translog function may not satisfy either monotonicity or convexity for certain input combinations (See Appendix A3.1 for the properties of translog function). Of the two remaining functions, VES functions are considered more flexible than CES functions because VES functions allow

the elasticity of substitution to change with different input combinations.

Although the translog function is the most flexible form among the candidate functions, it is perfectly plausible that a less generalized form provides a closer approximation of the gross payoff function than the translog function. Thus each of the candidate function is estimated below, and the functional form that explains the highest proportion of the variance in the response variable is selected as an approximation for the "true" production function. The only exception to this procedure is the Cobb Douglas form which is not estimated separately, because the Cobb Douglas hypothesis can be tested simultaneously with the estimation of the CES function.

CES Function: The gross payoff function in the CES form can be given as follows:

$$(5.6) Y = A [d D_a^{-n} + (1-d) D_c^{-n}]^{-h/n} + e;$$

where the parameters $A > 0$, $n \geq -1$, $0 < d < 1$, $h > 0$, and e an error term. A change in A changes the output for any given set of inputs in the same proportion, and is called the efficiency parameter. The elasticity of substitution of the CES function is given as $1/(1+n)$, and the parameter n is termed the substitution parameter. The parameter h determines the homogeneity of the production function, and is called the scale parameter. For any given value of n , the functional distribution of income is determined by d , the distribution parameter [Arrow1961].

Consistent estimates of the parameters of (5.6) can be obtained by Taylor series expansion [Kmenta 1967]. The CES function (5.6) can be written in the logarithmic form as follows:

$$(5.7) \ln Y = \ln A - h/n \ln [d D_a^{-n} + (1-d) D_c^{-n}] + u;$$

where u represents a classical error term. A Taylor series expansion of (5.7) around $n = 0$ leads to (5.8) if the third and higher order terms are ignored:

$$(5.8) \ln Y = \ln A + hd \ln D_a + h(1-d) \ln D_c \\ - \frac{1}{2} nhd(1-d) [\ln D_a - \ln D_c]^2 + u ;$$

Thus the estimates of the parameters of the CES function and their standard errors can be obtained from the following ordinary least squares regression:

$$(5.9) \ln Y = b_0 + b_1 \ln D_a + b_2 \ln D_c + b_3 [\ln D_a - \ln D_c]^2 + u ;$$

where $b_0 = \ln A$, $b_1 = hd$, $b_2 = h(1-d)$, $b_3 = -\frac{1}{2} nhd(1-d)$. The values of b_0 , b_1 , b_2 , and b_3 can be used to solve for A , d , n , and h . Equation (5.9) can also be used to test the hypothesis that the appropriate functional form is Cobb Douglas by examining the significance of the coefficient b_3 . A low significance level for b_3 indicates that the appropriate functional form is Cobb Douglas.

An OLS regression of (5.9) is given in (5.10). Note that the significance of the t-statistics for the coefficients are given parenthetically beneath the estimated values:

$$(5.10) \ln \hat{Y} = 0.338 + 0.206 \ln D_a + 0.367 \ln D_c \\ (0.000) \quad (0.000) \quad (0.000) \\ + .0402 [\ln D_a - \ln D_c]^2 ; \\ (0.008)$$

$$\text{SIGNIF} = 0.000; \quad R^2 = 0.514; \quad \text{Adjusted } R^2 = 0.499.$$

The high significance level of the coefficient of $[\ln D_a - \ln D_c]^2$ indicates that Cobb Douglas is not the appropriate functional form. The equivalent CES form of (5.10) can be obtained by solving for its parameters:

$A = 1.402$, $h = 0.573$, $d = .360$, and $n = - 0.609$.

The estimated CES function is highly significant, and its parameter estimates also lie within permissible limits. However, it fails to provide an adequate approximation to the gross payoff function as judged by its low R^2 and adjusted R^2 values.

VES Function: There are a few functional forms available that allow the elasticity of substitution to change with changing input combinations. However, only one such functional form, Sato-Revankar VES function, can be estimated directly from its input and output variables [Mukhopadhyay 1986]. This form of the gross payoff function can be given as [Revankar 1971]:

$$(5.11) Y = \gamma D_c^{\alpha(1-\theta\rho)} [D_a + (\rho-1) D_c]^{\alpha\theta\rho} + e;$$

where $\gamma > 0$, $\alpha > 0$, $0 < \theta < 1$, $0 \leq \theta\rho \leq 1$, $D_a / D_c > (1-\rho)/(1-\theta\rho)$, and e an error term. It can be verified that the elasticity of substitution σ is a linear function of the input proportion (D_c / D_a):

$$(5.12) \sigma = 1 + B (D_c / D_a) \text{ where } B = (\rho-1)/(1-\theta\rho).$$

Equation (5.11) can be estimated using a Taylor series approximation [Harvey 1977]. Dividing (5.11) by D_a and taking logarithm yields:

$$(5.13) \ln y = \ln \gamma + (\alpha-1)\ln D_a + \alpha\theta\rho \ln[1+(\rho-1)d] + \alpha(1-\theta\rho)\ln d + u$$

where $y = Y / D_a$, $d = D_c / D_a$, and u a classical disturbance term. A Taylor series expansion around $\rho = 1$ then leads to:

$$(5.14) \ln y = b_0 + b_1 \ln D_a + b_2 d + b_3 \ln d + u ;$$

where $b_0 = \ln \gamma$, $b_1 = (\alpha-1)$, $b_2 = \alpha\theta\rho (\rho-1)$, $b_3 = \alpha (1-\theta\rho)$. An OLS regression of (5.14) generates the following function:

$$(5.15) \ln \hat{y} = 0.313 - 0.432 \ln D_a + 0.034 d + 0.291 \ln d.$$

$$(0.000) \quad (0.000) \quad (0.034) \quad (0.000)$$

$$\text{SIGNIF} = 0.000; \quad R^2 = 0.883; \quad \text{Adjusted } R^2 = 0.879.$$

The estimated function (5.15) is highly significant, and also explains a high proportion of the variance in the response variable as evidenced by the high values of R^2 and adjusted R^2 . Moreover, the parameter estimates also lie within the permissible limits:

$$\gamma = 1.368; \alpha = 0.568; \rho = 1.123; \theta = 0.435.$$

Translog Function: The translog function for the two inputs D_a and D_c can be given as:

$$(5.16) \ln Y = a_0 + a_1 \ln D_a + a_2 \ln D_c + a_3 (\ln D_a)^2 + a_4 (\ln D_c)^2 \\ + a_5 \ln D_a \ln D_c + u;$$

An ordinary least squares regression of (5.16) produces the following:

$$(5.17) \ln \hat{Y} = - 0.132 - 0.061 \ln D_a - 0.629 \ln D_c + 0.018 (\ln D_a)^2 \\ (0.073) \quad (0.601) \quad (0.000) \quad (0.601) \\ - 0.313 (\ln D_c)^2 - 0.336 \ln D_a \ln D_c; \\ (0.000) \quad (0.000)$$

$$\text{SIGNIF} = 0.000; R^2 = 0.807; \text{Adjusted } R^2 = 0.797.$$

The estimated translog function (5.17) is definitely significant, but some of its estimated parameters (a_1 and a_3) are not. It is suspected that a_1 and a_3 are not significant due to a multicollinearity problem. However, the VES function (5.15), with fewer estimated parameters, provides a better approximation of the true gross payoff function as evidenced by its higher R^2 and adjusted R^2 values, and is selected as the final model.

Decision Generation Function: There are only two possible functional forms for the three variable decision generation function (5.5): (a) Cobb Douglas, and (b) Translog function, since neither the CES nor the VES function in three inputs can be estimated directly from their inputs and outputs by using the Taylor series method. Since the translog

function is an extension of the Cobb Douglas model, only the translog function is estimated below, and the Cobb Douglas hypothesis is tested by using a standard statistical procedure.

Translog Function: The estimated translog model is given below:

$$\begin{aligned}
 (5.18) \ln \hat{D}_a = & - 0.141 + 0.698 \ln R_a - 0.132 \ln Q_a - 0.167 \ln Q_c \\
 & (0.000) \quad (0.000) \quad (0.092) \quad (0.000) \\
 & - 2.555 (\ln R_a)^2 - 1.197 (\ln Q_a)^2 - 0.108 (\ln Q_c)^2 \\
 & (0.000) \quad (0.000) \quad (0.000) \\
 & + 0.180 \ln R_a \ln Q_a - 0.010 \ln R_a \ln Q_c - 0.643 \ln Q_a \ln Q_c \\
 & (0.146) \quad (0.769) \quad (0.000)
 \end{aligned}$$

SIGNIF = 0.000 ; $R^2 = 0.99089$; Adjusted $R^2 = 0.99018$.

The Cobb-Douglas alternative can be tested by examining the null hypothesis that all quadratic terms in (5.18) are equal to zero. The F-stat for this case is 133.156 which is much larger than $F_{6,115} = 2.96$ for alpha = 0.01 indicating rejection of the null hypothesis and the Cobb-Douglas alternative.

Although the translog function in (5.18) is significant, it exhibits multicollinearity as seen by the low significance levels of some of the parameters. Furthermore, the highest value of R^2 delete is 0.99088 indicating the presence of multicollinearity. The following methods may be used to deal with multicollinearity [Intriligator 1978, p.151]:

(1) Change sample: Since multicollinearity can be viewed as a sample problem, a popular method is to change the sample. However, there is no assurance that this approach will eliminate multicollinearity.

(2) Reduce the model: This method drops those variables that do not contribute to the explanatory power of the model. For a well specified model, such as the translog model, this approach introduces bias.

(3) Do nothing: If the purpose of the model is very elementary analysis, as is the case here, no action may be taken to lessen multicollinearity. Thus equation (5.18) will be considered the final model for the decision generation function.

5.6 Interpretation of Production Functions

The production functions estimated above can be used to understand the decision production process in fixed reorder cycle systems. For each production function, three important properties will be discussed in this section: (a) Marginal Products, (b) Substitution of Inputs, and (c) Returns to Scale. At this point, it should be recalled that all three properties are local characteristics of a production function, and thus depend on the specific input levels.

1. Gross Payoff Function: The functional form of the estimated VES function (5.15) is given as:

$$(5.19) \hat{Y} = 1.368 D_C^{0.281} [D_A + 0.123 D_C]^{0.277}.$$

The VES function (5.19) can be used to study the properties of the gross payoff function as follows:

(A) Marginal Products: First consider the input proportion $D_A = D_C$ [Figure 5.9]. Along this input ray, the ratio of marginal products of decision accuracy $[MP(D_A)]$ and coverage $[MP(D_C)]$ is given by:

$$(5.20) MP(D_A) / MP(D_C) = 0.768$$

Thus for a hypothetical firm with $D_A = D_C$, increasing decision coverage is more effective than increasing decision accuracy.

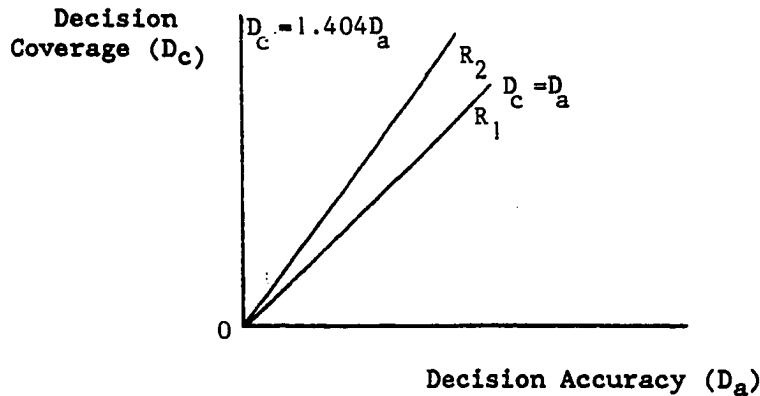
However, increasing decision coverage may not be more effective for all input combinations. To see this, calculate the ratio of marginal products for an arbitrary input combination:

$$(5.21) \quad MP(D_a) / MP(D_c) < 1 \quad \text{if } (D_c/D_a) < 1.404$$

$$> 1 \quad \text{if } (D_c/D_a) > 1.404.$$

In other words, increasing decision accuracy rather than decision coverage is more useful for any input proportion above the input ray OR_2 [Figure 5.9]. In summary, it is more effective to increase decision coverage rather than decision accuracy for a majority of input combinations. However, if decision coverage becomes too high compared to decision accuracy, it is more useful to increase decision accuracy rather than decision coverage.

FIGURE 5.9: INPUT COMBINATIONS OF GROSS PAYOFF FUNCTION



The interaction between the two inputs can be checked from the cross derivative of (5.19). It can be shown that the cross derivative is positive if $D_c/D_a < 5.464$. That is, the two inputs can be called complementary for most input combinations [Frisch 1956, p.60]. In other words, an increase in the value of one input should increase the marginal productivity of the other if $D_c/D_a < 5.464$.

(B) Elasticity of Substitution: The elasticity of substitution (σ) for the gross payoff function (5.19) can be calculated as follows:

$$(5.22) \quad \sigma = 1 + 0.240 (D_c/D_a) > 0.$$

The elasticity of substitution of the gross payoff function determines

the curvature of its isoquants. Two isoquants for the gross payoff function are illustrated in Figure 5.10.

The significance of the elasticity of substitution relation in (5.22) can be explained by comparing the VES isoquants with the hypothetical Cobb Douglas (CD) isoquants (with unit elasticity of substitution). There are two important differences between the two sets of isoquants. First, the VES isoquants are not as flat as the CD isoquants indicating that it is relatively easier to substitute decision accuracy for decision coverage, and vice versa. This property of the gross payoff function is due to the fact that the VES function exhibits a higher elasticity of substitution ($\sigma > 1$) for any input combination.

Second, the elasticity of substitution is fixed at unity for any input combination on the Cobb Douglas isoquants. The elasticity of substitution of the VES function, however, changes with the input ratio (D_c/D_a). For low values of the input ratio (D_c/D_a), the elasticity of the VES function approaches that of the Cobb Douglas function. However, for higher values of the input ratio, the two functions differ considerably. For high values of the input ratio, the VES function exhibits a significantly higher elasticity. In other words, it is easier to substitute as the input ratio is increased.

(C) Returns to Scale: The scale parameter (α) determines the homogeneity of the gross payoff function. An $\alpha < 1$ indicates decreasing returns to scale. Thus the scale parameter value of 0.568 implies decreasing returns to scale. For example, if both decision accuracy and coverage are increased by 10%, gross payoff increases only by 5.6%. In summary, the properties of the gross payoff function (5.19) clearly exhibit neoclassical properties.

2. Decision Generation Function: The estimated translog function (5.18) can be used to examine the decision production process.

(A) Marginal Products: It can be shown that the marginal products of the inputs are positive and diminishing in the relevant range of production. The rate of technical substitution (ratio of marginal products) for each input pair can be examined for the decision generation function. First consider the input pair Q_a and R_a :

$$(5.23) \frac{MP(Q_a)}{MP(R_a)} = \frac{-0.132 + 0.180 \ln R_a - 2.394 \ln Q_a - 0.643 \ln Q_c}{0.698 - 5.111 \ln R_a + 0.180 \ln Q_a - 0.010 \ln Q_c} \frac{R_a}{Q_a}$$

Equation (5.23) indicates that the ratio of the two marginal products depends on the values of all three variables. It can be seen from Figure 5.11 that $MP(Q_a) < MP(R_a)$ for low values of R_a/Q_a . But as R_a/Q_a increases, $MP(Q_a)/MP(R_a)$ increases, and eventually $MP(Q_a)$ may exceed $MP(R_a)$. In other words, when R_a is low compared to Q_a , it is more effective to increase R_a . However, as R_a becomes high compared to Q_a , it becomes more useful to increase Q_a . This transition point occurs earlier with lower Q_c .

Similar results hold for two other pair of inputs: (R_a, Q_c) and (Q_a, Q_c) . That is, it is more useful to increase an input when it has a low value. The relative effect of an input reduces as it is increased. Often, after a threshold point, it is no longer effective to increase the same input.

The marginal rate of technical substitution of any two inputs has an inverse relationship with the third input. For example, $MP(Q_a)/MP(R_a)$ decreases with Q_c . Similarly, for input pair (R_a, Q_c) , $MP(Q_c)/MP(R_a)$ decreases with Q_a , and for (Q_a, Q_c) , $MP(Q_c)/MP(Q_a)$ diminishes with increasing R_a .

FIGURE 5.10: ISOQUANTS OF GROSS PAYOFF FUNCTION

Curve 1 and 2: Isoquants of gross payoff function

Curve 3 and 4: Isoquants of Cobb-Douglas function

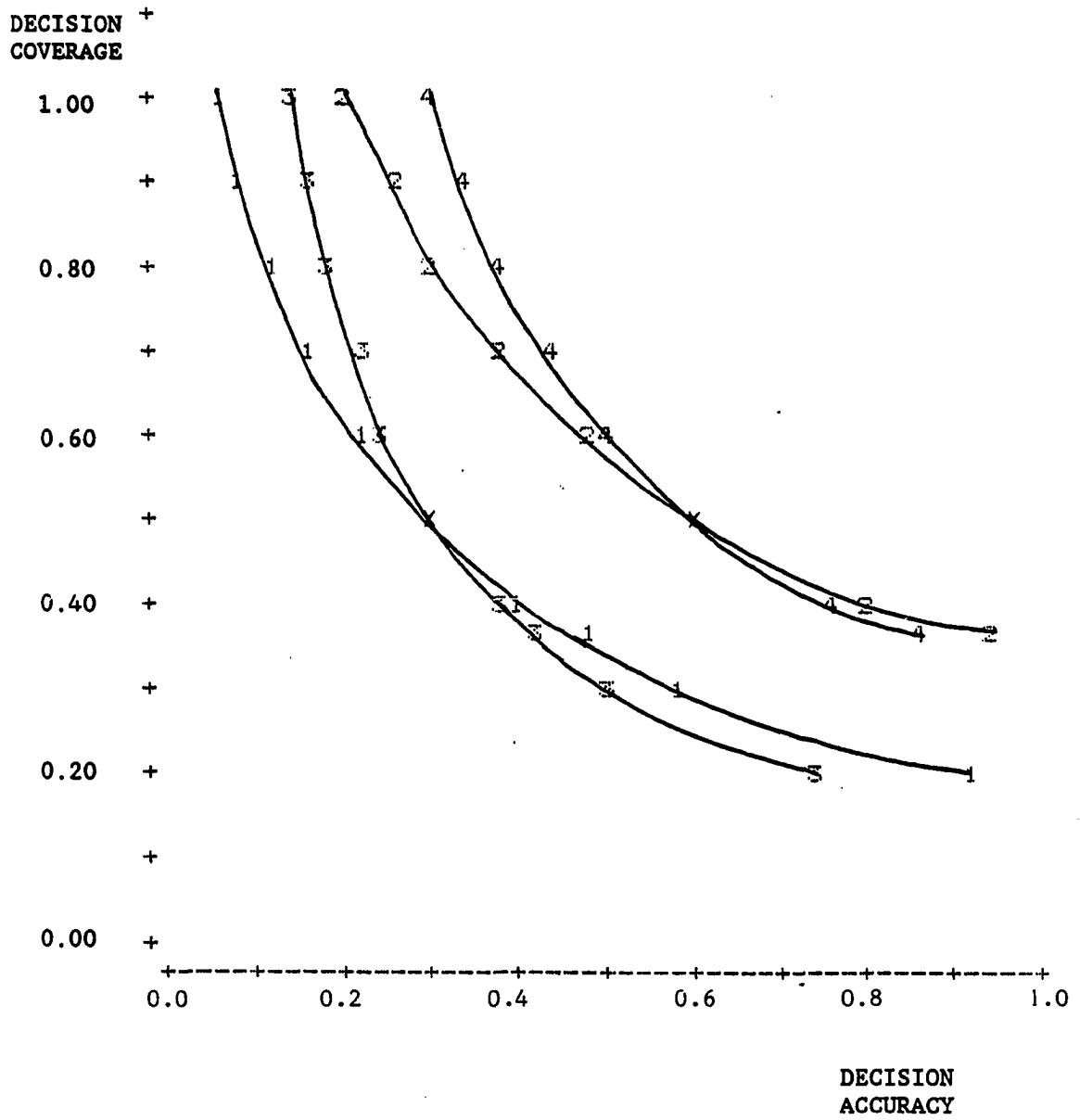
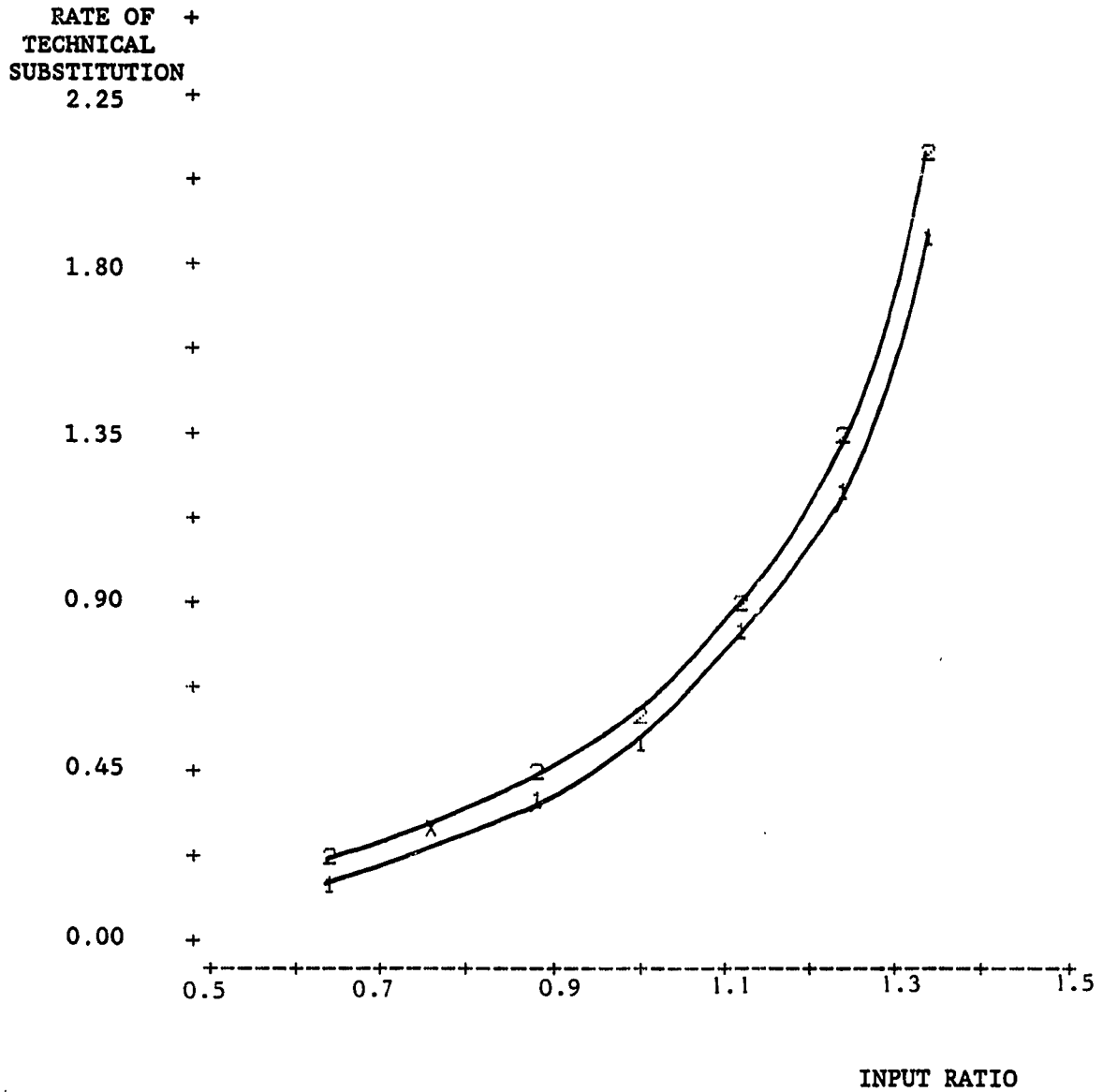


FIGURE 5.11: RATE OF TECHNICAL SUBSTITUTION BETWEEN ORDER-UP-TO POINT AND QUANTITY ON RECORD ACCURACY (R_a & Q_a)

Vertical axis: Marginal Product (Q_a) / Marginal Product (R_a)
 Horizontal axis: Input ratio (R_a / Q_a)

Curve 1: Quantity on record accuracy = 0.50
 Quantity on record coverage = 0.33

Curve 2: Quantity on record accuracy = 0.50
 Quantity on record coverage = 0.25



The interaction between each pair of inputs can be examined from the cross derivatives of (5.18). The cross derivatives with respect to $(R_a$ and $Q_a)$ and $(R_a$ and $Q_c)$ can be shown to be positive in the relevant range indicating that the inputs in each pair are complements to each other. However, it can be shown that the cross derivative of D_a with respect to Q_a and Q_c is negative except for low values of Q_a and Q_c . To see this, differentiate (5.18) with respect to Q_a and Q_c to obtain:

$$(5.24) \quad \frac{\delta^2 D_a}{\delta Q_a \delta Q_c} = \frac{D}{Q_a Q_c} Q'_a Q'_c$$

where $Q'_a = [-0.132 + 0.180 \ln R_a - 2.394 \ln Q_a - 0.643 \ln Q_c]$

$$Q'_c = [-0.167 - 0.010 \ln R_a - 0.643 \ln Q_a - 0.216 \ln Q_c]$$

It can be seen that along the input ray $R_a = Q_a = Q_c$, this cross derivative is negative for all $R_a, Q_a, Q_c > 0.61$. That is, the marginal product of Q_c diminishes with increasing Q_a and vice versa. In other words, Q_a and Q_c can be called competitive inputs for $R_a, Q_a, Q_c > 0.61$.

(B) Elasticity of Substitution: The elasticity of substitution between an input pair x_m and x_n of an arbitrary translog function

$\ln Y = a_0 + \sum_m a_m \ln x_m + \sum_m \sum_n a_{mn} \ln x_m \ln x_n$; $m, n = 1, 2, 3$ and $m \leq n$; can be given as [see Appendix A3.1]:

$$\sigma_{mn} = \frac{x'_m + x'_n}{2a_{mn} + x'_m + x'_n - 2a_{mm}(x'_n/x'_m) - 2a_{nn}(x'_m/x'_n)}; \quad x'_p = MP(x_i) \frac{x_i}{Y}$$

Using this formula, it can be shown that the elasticity of substitution between any two inputs is positive. The properties of the elasticity of substitution for each input pair are discussed below:

(i) R_a and Q_a (σ_{12}): The elasticity of substitution between these two inputs does not exceed unity. To see this let R_a be input 1 (x_1) and Q_a input 2 (x_2). Since $x'_p > 0$ ($p = 1, 2$), $\sigma_{12} > 1$ if and only if

$$a_{12} - a_{11}(x'_2/x'_1) - a_{22}(x'_1/x'_2) < 0.$$

From (5.18) it is clear that $a_{12} > 0$, a_{11} , $a_{22} < 0$ so that this condition can never be true. The value of σ_{12} for any input combination depends on the specific values of all three inputs. However, it can be shown that σ_{12} decreases with increasing Q_a and Q_c . It also decreases with increasing R_a except for high values of Q_c .

(ii) R_a and Q_c (σ_{13}): It can be easily shown that $\sigma_{13} < 1$. Denoting R_a as input 1 (x_1) and Q_c as input 3 (x_3), the condition for $\sigma_{13} > 1$ can be given as:

$$-0.010 + 2.555 (x_3'/x_1') + 0.108 (x_1'/x_3') < 0.$$

Denoting $x_3'/x_1' = u (>0)$, it can be easily verified that the minimum value of $v = -0.010 + 2.555 u + 0.108 / u > 0$. The value of σ_{13} also depends on the specific values of R_a , Q_a and Q_c . However, it can be shown that σ_{13} decreases with increasing R_a and Q_c . It also decreases with increasing Q_a except for high values of R_a .

(iii) Q_a and Q_c (σ_{23}): The properties of this elasticity are very similar to those of σ_{12} and σ_{23} . It also does not exceed unity and decreases with the increase of any of the three inputs.

It is also found that $\sigma_{12} < \sigma_{13} < \sigma_{23}$. Figure 5.12 illustrates the relative magnitudes of the three elasticities for certain input combinations.

(C) Returns to Scale: The condition for increasing returns to scale can be derived using equation (A3.6):

$$(5.25) - 4.940 \ln R_a - 2.857 \ln Q_a - 1.013 \ln Q_c > 0.601.$$

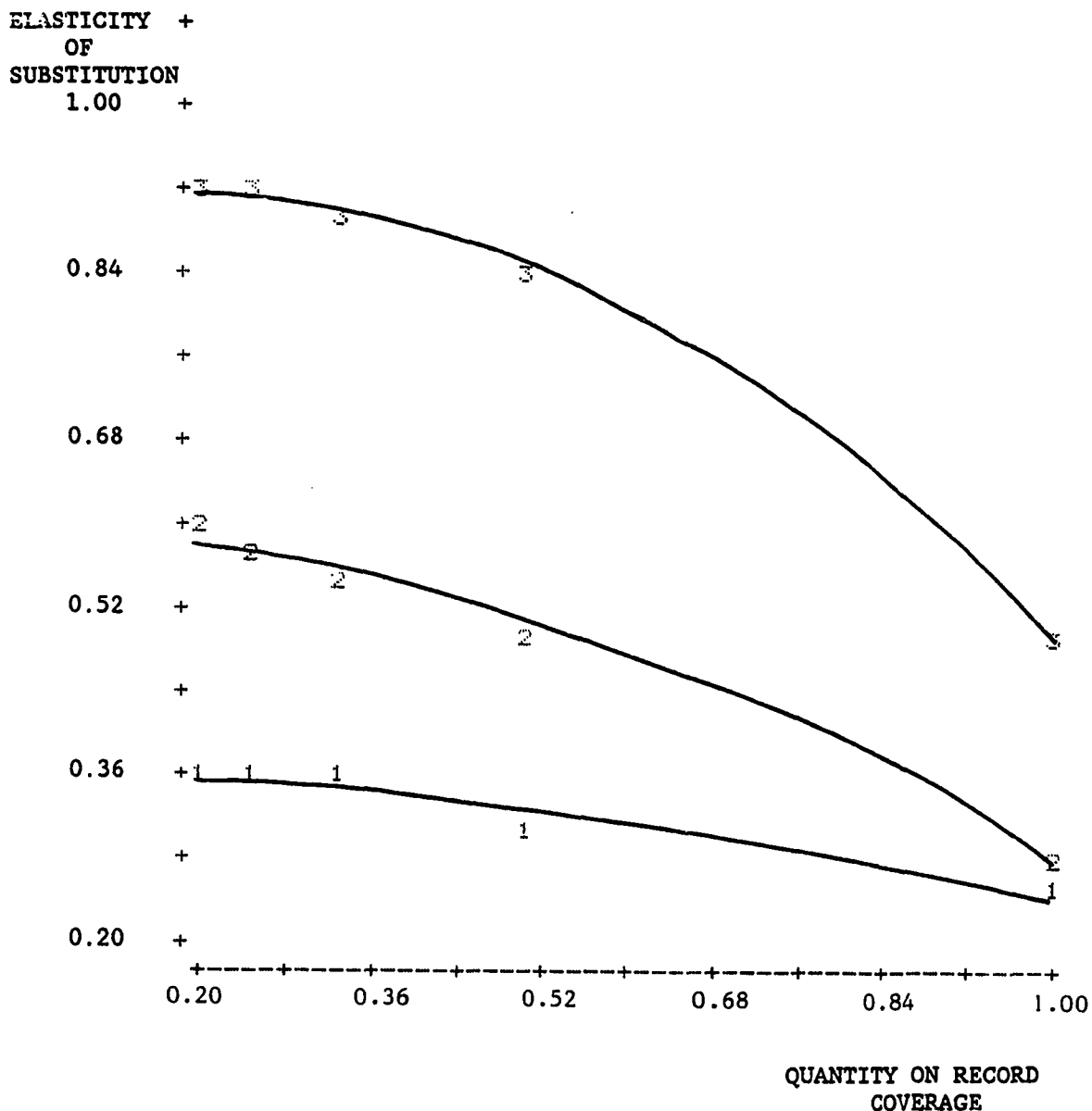
Equation (5.25) indicates that the decision generation function exhibits increasing returns to scale for most input combinations. However, for very high values of R_a , Q_a and Q_c (for example, $R_a = Q_a = Q_c = 0.934$), decreasing returns to scale sets in.

FIGURE 5.12: A COMPARISON OF ELASTICITIES OF SUBSTITUTION
 [ORDER-UP-TO POINT ACCURACY = 0.68 & QUANTITY ON RECORD ACCURACY = 0.68]

Curve 1: Elasticity of substitution between
 Order-up-to point accuracy and Quantity on record accuracy

Curve 2: Elasticity of substitution between
 Order-up-to point accuracy and Quantity on record coverage

Curve 3: Elasticity of substitution between
 Quantity on record accuracy and Quantity on record coverage



5.7 Backordering Case

The simulation is redesigned for the backordering case with a supply lead time of four days. The modified simulation model is designed in much the same way as the original model. Based on four pilot runs, the initial cut off point is fixed at 50th decision period. The method of antithetic variates is used for variance reduction, and a stepwise strategy is used to select the appropriate values of run length and number of replications.

The simulation is run in two parts to study the decision production model. First the model is used to generate total product curve for each input. The shape of these curves are very similar to Figure 5.1 through 5.6, and for the sake of brevity are not presented here. Thus the experimental design used for this case is the same used for the lost sales case. The estimated production functions are discussed below.

1. Gross Payoff Function: The VES function is selected as the appropriate functional form based on its R^2 and adjusted R^2 values:

$$(5.26) \ln \hat{y} = 0.142 - 0.675 \ln D_a + 0.046 d + 0.102 \ln d$$

$$(0.000) \quad (0.000) \quad (0.007) \quad (0.000)$$

$$\text{SIGNIF} = 0.000; \quad R^2 = 0.947; \quad \text{Adjusted } R^2 = 0.945.$$

The functional form of the estimated function is given by:

$$(5.27) \hat{Y} = 1.153 D_c^{0.102} [D_a + 0.207 D_c]^{0.314}.$$

The properties of the gross payoff function (5.27) is discussed briefly in the following paragraphs:

(A) Marginal products: It can be easily verified that the marginal products of (5.27) are positive and diminishing in the relevant range of production. The ratio of the marginal products for an arbitrary input combination is given by:

$$(5.28) \quad MP(D_a) / MP(D_c) < 1 \text{ if } (D_c/D_a) < 0.653 \\ > 1 \text{ if } (D_c/D_a) > 0.653.$$

A significant difference compared to the lost sales case [see equation (5.21)] is that $MP(D_a)$ exceeds $MP(D_c)$ at a much smaller input ratio D_c/D_a . It can also be shown that the two inputs are complements for any input combination with $D_c/D_a < 0.730$. Compared to the lost sales case, the two inputs are competitive for a larger input space.

(B) Elasticity of Substitution: The elasticity of substitution (σ) between the two inputs is given by:

$$(5.29) \quad \sigma = 1 + 0.659 (D_c/D_a).$$

Once again, this elasticity is greater than unity. Moreover, the elasticity is higher than the lost sales case indicating greater ease in substituting one input for another.

(C) Returns to Scale: Decreasing returns to scale is evidenced by the scale parameter $\alpha = 0.325$. In fact the scale parameter is lower compared to the lost sales case.

2. Decision Generation Function: The following translog model is found to be suitable for the decision generation function [see Appendix A3.4 for details of the estimated model]:

$$(5.30) \quad \ln \hat{D}_a = -0.182 + 0.411 \ln R_a - 0.161 \ln Q_a - 0.308 \ln Q_c \\ (0.000) \quad (0.000) \quad (0.037) \quad (0.000) \\ + 0.022 (\ln R_a)^2 - 0.282 (\ln Q_a)^2 - 0.225 (\ln Q_c)^2 \\ (0.251) \quad (0.001) \quad (0.000) \\ -0.092 \ln R_a \ln Q_a + 0.048 \ln R_a \ln Q_c - 0.554 \ln Q_a \ln Q_c \\ (0.007) \quad (0.005) \quad (0.000)$$

$$\text{SIGNIF} = 0.000; \quad R^2 = 0.97770; \quad \text{Adjusted } R^2 = 0.97595.$$

This translog model is highly significant and provides excellent fit

for the decision generation function. The Cobb Douglas alternative is rejected because the null hypothesis that the second order terms are zero is rejected at $\alpha = 0.01$. Although the estimated function exhibits multicollinearity, for reasons stated in Section 5.5, equation (5.30) is used to examine the properties of the decision generation function:

(A) Marginal Products: It can be easily verified that the marginal products of the inputs are positive and diminishing in the relevant range of production. The rate of technical substitution between each input pair can be examined from the ratio of respective marginal products. Figure 5.13, for example, illustrates the rate of technical substitution between Q_a and R_a ($MP(Q_a) / MP(R_a)$) for two values of Q_c . It can be seen that for low values of R_a/Q_a , it is more effective to increase R_a since $MP(R_a) > MP(Q_a)$. However, after a threshold level it is more useful to increase Q_a rather than R_a . Similar results hold for two other input pairs: (R_a, Q_c) and (Q_a, Q_c) .

It is also found that the marginal rate of technical substitution of any two inputs decreases with the third input. For example, $MP(Q_a/R_a)$ decreases with Q_c .

The interaction between each input pair can be studied from the cross derivatives of (5.30). The cross derivatives with respect to (R_a, Q_a) and (R_a, Q_c) are positive in the relevant range indicating that inputs in these pairs are complementary. However, the cross derivative with respect to (Q_a, Q_c) becomes negative for high values of Q_a , implying that this input pair becomes competitive if Q_a is high.

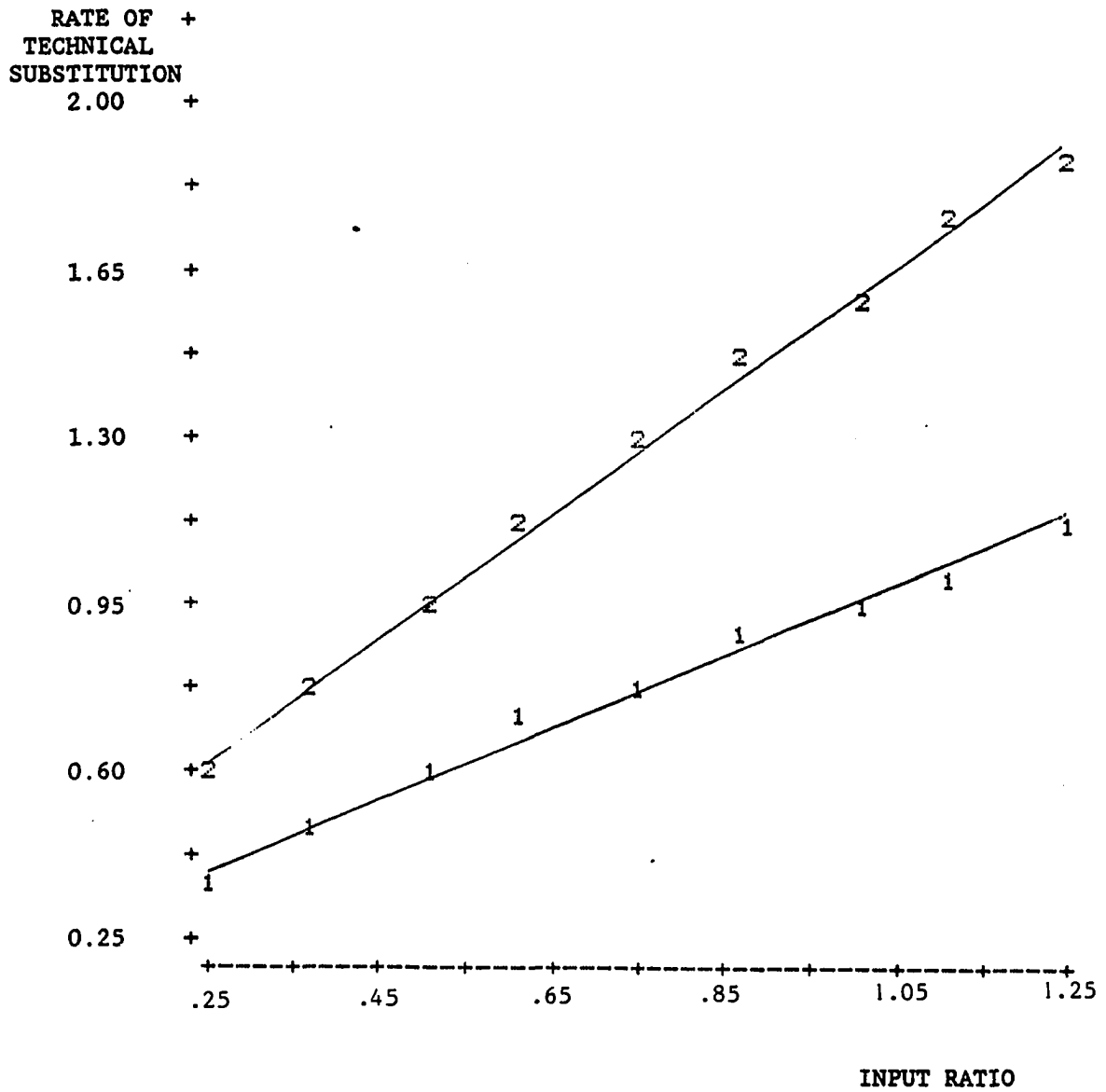
(B) Elasticity of substitution: It can be shown that the elasticity of substitution between any two inputs is positive. The properties of the elasticity of substitution for each input pair are discussed below.

FIGURE 5.13: RATE OF TECHNICAL SUBSTITUTION BETWEEN ORDER-UP-TO POINT AND QUANTITY ON RECORD ACCURACY (R_a & Q_a)

Vertical axis: Marginal Product (Q_a) / Marginal Product (R_a)
 Horizontal axis: Input ratio (R_a / Q_a)

Curve 1: Quantity on record accuracy = 0.80
 Quantity on record coverage = 0.50

Curve 2: Quantity on record accuracy = 0.80
 Quantity on record coverage = 0.33



(i) R_a and Q_a (σ_{12}): It can be shown that σ_{12} may exceed unity for certain input combinations. Moreover, its value decreases if any of the inputs is increased.

(ii) R_a and Q_c (σ_{13}): σ_{13} does not exceed unity and it has an inverse relation with Q_a and Q_c .

(iii) Q_a and Q_c (σ_{23}): σ_{23} can be greater than unity for certain input combinations, and also has an inverse relation with Q_a and Q_c .

It is also found that $\sigma_{13} < \sigma_{12}$ and $\sigma_{13} < \sigma_{23}$. But no such relation is found between σ_{12} and σ_{23} .

(C) Returns to Scale: The condition for increasing returns to scale for (5.30) is given by:

$$(5.31) - 0.0002 \ln R_a - 1.210 \ln Q_a - 0.956 \ln Q_c > 1.131.$$

Equation (5.31) indicates that the decision generation function exhibits increasing returns to scale for low levels of inputs. However, as input levels are increased, decreasing returns to scale sets in. For example, (5.30) exhibits decreasing returns to scale for $R_a, Q_a, Q_c > 0.60$.

A significant difference between the decision generation functions for the lost sale case and backordering case lies in their returns to scale property. Whereas the decision generation function for the lost sales case exhibits decreasing returns for a small part of the input space (high values of Q_a and Q_c), the backordering case is subject to decreasing returns for a considerable part of the input space.

5.8 Summary

A simulation model was used in this chapter to study fixed reorder cycle systems. The results of the simulation are in general agreement with the results obtained in Chapter 4 using the analytical method. Both

methods affirm the neoclassical characteristics of the decision production model. The simulation method, however, extends and generalizes the results obtained by the analytical method.

The efficacy of the decision production model can also be appreciated from the results obtained in this chapter. A great deal of understanding of the fixed reorder cycle system can be attained by studying the properties of the gross payoff and decision generation functions. Some of the important properties such as the rate of technical substitution, elasticity of substitution, and returns to scale were examined in the preceding sections.

The results for the two cases, lost sales case and backordering case, are very similar with some subtle differences. The gross payoff function for both cases exhibit decreasing returns to scale and elasticity of substitution greater than unity. However, the backordering case is characterized by lower returns to scale and higher elasticity of substitution compared to the lost sales case.

The decision generation function for both the lost sales and backordering case also exhibit very similar properties. However, one significant difference between the two cases is seen in their returns to scale property. While the lost sales case is characterized by decreasing returns to scale only for a limited part of the input space (very high values of inputs), the backordering case displays decreasing returns to scale for a considerable portion of the input space.

The simulation model used in this study enabled a comprehensive study of the fixed reorder cycle information system. The implications of this study and some directions for future research will be discussed in the following chapter.

CHAPTER 6
CONCLUSION

In response to the need to estimate the effects of MIS on firm productivity, a microeconomic approach is developed for MIS evaluation. The proposed approach is discussed in Chapter 3, and is illustrated and validated in Chapters 4 and 5 in the context of a fixed reorder cycle MIS. The purpose of this chapter is to discuss the implications of the results obtained in the previous chapters, and propose some directions for future research.

6.1 Implications of the Study

The implications of this study are examined in three parts. First, the uses of the proposed approach of MIS evaluation are discussed. Next, results from the evaluation of fixed reorder cycle systems are briefly reviewed. Finally, the significance of this study for MIS evaluation is analyzed.

Uses of the Proposed Approach: The proposed approach has both descriptive and normative uses. As a descriptive tool, it enables managers to systematically generate MIS design options, and examine the effects of each alternative on decisions as well as on firm output (gross payoff). As a normative tool, it allows managers to compare various design options based on their effects on firm productivity. The fixed reorder cycle MIS example can be used to illustrate the usage of

the proposed approach.

Two types of MIS design alternatives can be generated using this approach. The first type does not alter information accuracy or coverage, and thus gross payoff does not change. However, a specific design alternative may have a positive effect on firm productivity if it leads to a reduction in magnetic storage space, computer processing time, etc. The criteria of technical, allocative, and economic efficiencies of Information Generation can be used to compare MIS designs of this type.

The second type of design alternative leads to increased information accuracy and / or coverage, and thus results in improved decision accuracy. For example, order-up-to point coverage (R_c) may be improved by increasing its updating frequency, or quantity on record coverage (Q_c) may be enhanced by reducing its age. Although an increase in information accuracy or coverage is achieved at the expense of added resource usage, it is expected to improve decision accuracy, and hence gross payoff. The resulting effect on firm productivity is determined by the change in net payoff. If net payoff increases, firm productivity improves, otherwise productivity declines.

The tradeoffs between various MIS design alternatives can also be checked using the proposed approach. Consider, for example, a plan to increase order-up-to point accuracy (R_a) against the alternative to enhance quantity on record accuracy (Q_a). Since the two alternatives have different cost implications, the estimated Decision Generation function should be used to study the tradeoffs between them. Similarly, substitution possibilities between other input pairs such as R_a and Q_c , Q_a and Q_c , etc. should be checked.

At gross payoff level, both decision accuracy and coverage (D_a and D_c) can be changed to obtain desired effects on gross payoff. The effects of increasing decision accuracy and / or decision coverage and the possible tradeoffs between them can be studied using the gross payoff function.

Evaluation of Fixed Reorder Cycle MIS: The proposed approach has been operationalized and validated in the context of a fixed reorder cycle MIS. Some important results from this part of the research are discussed below.

At the gross payoff level, increasing decision coverage (D_c) is found to be more effective compared to increasing decision accuracy (D_a), except for high values of the input ratio (D_c/D_a). The elasticity of substitution between these two inputs exceeds unity implying that it is relatively easy to substitute decision accuracy for decision coverage, and vice versa.

All inputs of the Decision Generation function (except order-up-to point coverage) have substantial positive but declining effects on decision accuracy. Of all input pairs, quantity on record accuracy and coverage behave as competitive inputs for high values of both inputs. An increase in one diminishes the marginal product of the other input. Finally, the Decision Generation function is found to exhibit increasing returns to scale for most input combinations implying that a certain percentage increase in all input values results in a more than proportionate increase in decision accuracy.

The results obtained in this part of the study also make a contribution to the inventory control literature. The focus of the research in the inventory control literature has been to formulate

procedures for the determination of order-up-to point and review period, and not on the effects of information accuracy and coverage on fixed reorder cycle systems. Thus, this research contributes to the inventory control literature by examining the effects of MIS on inventory control decisions and firm output.

Significance for MIS Evaluation: The neoclassical viewpoint used and validated in this research provides valuable guidelines for MIS evaluation. First, the process of building a model for MIS evaluation is facilitated by procedures for the identification of inputs and outputs. Data, information, and decisions as inputs and outputs of this model are defined, and a metric for their measurement is developed. The neoclassical guidelines for the identification and measurement of resources as inputs can be used to complete model specification. Next, a variety of production functions and their estimation procedures from the econometric literature can be used in the model estimation stage. Finally, the criteria of relative efficiency, effectiveness, and productivity proposed in this research can be used to establish a weak ordering of any set of MIS design alternatives.

The neoclassical viewpoint taken in the proposed approach also restricts its applicability to structured management or operational control decisions characterized by high volume. However, this is a minor limitation, because the majority of MIS are used for routine and repetitive decisions only.

In summary, a first step is taken in this study to estimate the effects of MIS on firm productivity. In the face of increasing reliance on MIS for managerial decision making, continued research efforts are required for MIS evaluation.

6.2 Directions for Future Research

Further research is required in at least four specific directions to augment the work done in this dissertation:

1. Application of the Decision Production Model: The two stage decision production model has been illustrated in the context of one inventory control system. The decision production model can also be used to study the effects of MIS on many other systems. So long as the decision making context is routine and repetitive and is characterized by high volume, the decision production model can be used to compare alternative MIS. First, the decision production framework can be used to examine possible ways of improving the efficiency of Information Generation. Second, the effects of increasing information accuracy and coverage can be studied by using the decision generation function. Finally, the substitution possibilities between decision accuracy and coverage, and their effects on gross payoff can also be analyzed from the gross payoff function.

2. Extension of the Decision Production Model: The proposed framework is applicable to structured decisions involving operation and management control activities. Although most computer based MIS are in this category, efforts have been made in recent years to develop information systems to support less structured decisions. Thus a logical extension of the proposed model should encompass semistructured and unstructured decision contexts involving operation control, management control or strategic planning activities.

3. Evaluation of Organizational Information Systems: The proposed approach is applicable to information systems involving a single decision context. Often management is interested in evaluating the

overall organizational information system. Although an organizational information system can be viewed as an aggregate of several MIS the evaluation of an organizational information system requires special care. The proposed model is of little value in this context because of the complexity involved in an organizational information system. Thus a modified model should be developed to enable the aggregation of several interrelated management information systems.

A second modification of the proposed model should consider information systems that are primarily used for the production and distribution of information for external agencies such as customers, suppliers, investors, and government agencies. Such a modified approach would identify the criteria to be used for the evaluation of information systems for external users.

4. Link between Design and Evaluation: The relation between the design and evaluation of MIS has not been addressed explicitly in this research. Therefore, a future research project may be undertaken to combine the proposed evaluation technique with system analysis and design techniques. Some research questions to be addressed by such a research project would include the following: What are the possible methods of increasing information accuracy and coverage, and what are their effects on firm productivity? What are the possible enhancements to an existing MIS, and which alternative will have maximum impact on firm productivity?

In short, the proposed decision generation model can be used to evaluate MIS used for routine and repetitive decision contexts. Further research can be undertaken to extend the proposed approach to less structured decision contexts, organizational information systems, etc.

APPENDICES

APPENDIX 1:
ANALYTICAL SOLUTION

Al.1 Marginal Products of Inputs: The derivation of equations (4.21)

through (4.24) is given below.

Since $D_a = 1 - [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2} / d$ from (4.20),

$$\frac{\delta D_a}{\delta Q_a} = \frac{q^2 (1 - Q_a)}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{1/2}} > 0 \quad (4.21)$$

$$\begin{aligned} \frac{\delta^2 D_a}{\delta Q_a^2} &= \frac{q^2 - [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2} + q^2 (1-Q_a)^2 [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{-1/2}}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]} \\ &= \frac{q^2 - [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2] + q^2 (1-Q_a)^2}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} \\ &= - \frac{q^2 r^2 (1 - R_a)^2}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} < 0 \quad (4.22) \end{aligned}$$

Similarly, $\frac{\delta D_a}{\delta R_a} = \frac{r^2 (1 - R_a)}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{1/2}} > 0 \quad (4.23);$

$$\begin{aligned} \frac{\delta^2 D_a}{\delta R_a^2} &= \frac{r^2 - [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2} + r^2 (1-R_a)^2 [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{-1/2}}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]} \\ &= \frac{r^2 - [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2] + r^2 (1 - R_a)^2}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} \\ &= - \frac{r^2 q^2 (1 - Q_a)^2}{d [r^2 (1 - R_a)^2 + q^2 (1 - Q_a)^2]^{3/2}} < 0 \quad (4.24). \end{aligned}$$

Al.2 Elasticity of Substitution (S): The derivation of (4.28) is

given below. From (4.25):

$$S = \frac{d \ln(m)}{d \ln(n)} = \frac{n}{m} \frac{dm}{dn} \quad \text{where } m = \frac{Q_a}{R_a} \text{ and } n = \frac{r^2 (1-R_a)}{q^2 (1-Q_a)}.$$

$$\text{Or, } S = \frac{g_1 (g_1 R_a + g_2 Q_a)}{g_2 R_a Q_a (g_1 n_2 - g_2 n_1)}$$

where g_1 and g_2 are partial derivatives of the decision production function (4.20) and n_1 and n_2 are partial derivatives of n with respect to R_a and Q_a [see Henderson and Quandt 1980, p.73].

$$\text{Now } g_1 (g_1 R_a + g_2 Q_a) = \frac{r^2 (1-R_a) [r^2 (1-R_a) R_a + q^2 (1-Q_a) Q_a]}{d^2 [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]} .$$

$$\text{and } g_2 R_a Q_a (g_1 n_2 - g_2 n_1) = \frac{R_a Q_a r^2 (1-R_a) [r^2 (1-R_a) + q^2 (1-Q_a)]}{d^2 [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]}$$

$$\text{so that } S = \frac{\frac{q^2 (1-Q_a)}{1-R_a} + \frac{r^2 (1-R_a)}{1-Q_a}}{\frac{R_a}{q^2 (1-Q_a)} + \frac{Q_a}{r^2 (1-R_a)}} \geq 0.$$

A1.3 Returns to Scale: The derivation of equations (4.34) and (4.36) are explained here. The decision production function (4.20)

$$D_a = 1 - [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2} / d = g(R_a, Q_a)$$

exhibits increasing returns to scale if and only if

$$g(\lambda R_a, \lambda Q_a) > g(R_a, Q_a)$$

$$\text{or } 1 - [r^2 (1-\lambda R_a)^2 + q^2 (1-\lambda Q_a)^2]^{1/2} / d$$

$$> 1 - \lambda [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2} / d$$

$$\text{or } d - [r^2 (1-\lambda R_a)^2 + q^2 (1-\lambda Q_a)^2]^{1/2}$$

$$> \lambda d - \lambda [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2}$$

$$\text{or } d - \lambda d + \lambda [r^2 (1-R_a)^2 + q^2 (1-Q_a)^2]^{1/2}$$

$$> [r^2 (1-\lambda R_a)^2 + q^2 (1-\lambda Q_a)^2]^{1/2}$$

$$[\lambda^2 r^2 (1-R_a)^2 + \lambda^2 q^2 (1-Q_a)^2]^{1/2} - (\lambda-1)d$$

$$> [(\lambda r - \lambda r R_a)^2 + (\lambda q - \lambda q Q_a)^2]^{1/2}$$

$$> [(\lambda r - \lambda r R_a + r - \lambda r)^2 + (\lambda q - \lambda q Q_a + q - \lambda q)^2]^{1/2}$$

$$> [(\lambda r (1 - R_a) - r (\lambda - 1))^2 + (\lambda q (1 - Q_a) - q (\lambda - 1))^2]^{1/2}$$

$$\text{Now let } \Sigma_r = \lambda r (1-R_a),$$

$$\Sigma_q = \lambda q (1-Q_a),$$

$$D = (\lambda-1) d,$$

$$R = (\lambda - 1) r,$$

$$Q = (\lambda - 1) q.$$

Then the condition for increasing returns can be given as:

$$(\Sigma_r^2 + \Sigma_q^2)^{1/2} - D > [(\Sigma_r - R)^2 + (\Sigma_q - Q)^2]^{1/2}$$

Note that only positive values of square roots are relevant in the above inequality. Therefore, by squaring each side the inequality becomes:

$$\Sigma_r^2 + \Sigma_q^2 + D^2 - 2D(\Sigma_r^2 + \Sigma_q^2)^{1/2} > (\Sigma_r - R)^2 + (\Sigma_q - Q)^2$$

$$\text{or } \Sigma_r^2 + \Sigma_q^2 + D^2 - 2D(\Sigma_r^2 + \Sigma_q^2)^{1/2} > \Sigma_r^2 + R^2 + \Sigma_q^2 + Q^2 - 2R\Sigma_r - 2Q\Sigma_q$$

$$\text{or } D^2 - 2D(\Sigma_r^2 + \Sigma_q^2)^{1/2} > R^2 + Q^2 - 2R\Sigma_r - 2Q\Sigma_q$$

$$\text{or } D(\Sigma_r^2 + \Sigma_q^2)^{1/2} < \Sigma_r R + \Sigma_q Q - C;$$

where $C = (R^2 + Q^2 - D^2)/2 > 0$. Square each side of this inequality to obtain the following:

$$D^2 (\Sigma_r^2 + \Sigma_q^2) < \Sigma_r^2 R^2 + \Sigma_q^2 Q^2 + C^2 + 2RQ\Sigma_r\Sigma_q - 2CR\Sigma_r - 2CQ\Sigma_q$$

Upon simplification, the condition becomes:

$$(4.34) \quad \Sigma_r^2 (R^2 - D^2) + C^2 + \Sigma_q^2 (Q^2 - D^2) \\ + \Sigma_r R (\Sigma_q Q - 2C) + \Sigma_q Q (\Sigma_r R - 2C) > 0.$$

Since $R > D$, the first two terms in (4.34) must be positive. The third term also becomes positive if the review period (T) is smaller than the replenishment lead time (L) such that $Q > D$. The conditions for the last two terms becoming positive are derived in the following way. The fourth term becomes positive if $\Sigma_q Q - 2C > 0$. Substitute for Σ_q and C to obtain the following inequality:

$$\lambda q (1 - Q_a) Q > R^2 + Q^2 - D^2$$

$$\text{or } (Q + q) (1 - Q_a) Q > R^2 + Q^2 - R^2 - Q^2 + 2RQ \quad \text{since } D = R - Q$$

$$\text{or } 1 - Q_a > 2R / (Q + q) \quad \text{or } Q_a < 1 - 2R / (Q + q).$$

Thus the conditions for the last two terms in (4.34) being positive are:

$$(4.36) \quad R_a < 1 - \frac{2Q}{(R + r)} \quad \text{and} \quad Q_a < 1 - \frac{2R}{(Q + q)}.$$

A1.4 Numerical Example for the Backordering Case:

For a review period of $T = 6$, the accurate magnitudes of order-up-to point and quantity on record information can be determined by examining the decision problem faced by a rational manager with accurate knowledge of demand and stock information:

$$\begin{aligned} \text{Maximize: } 6N_t &= C_0 (X_1 + X_2 + X_3 + X_4 + X_5 + X_6) \\ &- C_1 (I_t - X_1 - C_2)^2 \\ &- C_1 (I_t - X_1 - X_2 - C_2)^2 \\ &- C_1 (I_t - X_1 - X_2 - X_3 - C_2)^2 \\ &- C_1 (I_t - X_1 - X_2 - X_3 - X_4 - C_2)^2 \\ &- C_1 (I_t - X_1 - X_2 - X_3 - X_4 - X_5 - C_2)^2 \\ &- C_1 (I_t - X_1 - X_2 - X_3 - X_4 - X_5 - X_6 - C_2)^2 \end{aligned}$$

where I_t is the starting inventory of the review period. The solution of the above problem is $I_t = 1/6[6X_1 + 5X_2 + 4X_3 + 3X_4 + 2X_5 + X_6] + C_2$. The expected value of I_t is given by $E[I_t] = 3.5\mu + C_2$ because $E[X_i] = \mu$; $i = 1, 2, \dots, 6$. Since $E[I_t]$ reflects the expected value of the quantity on hand in the beginning of the review period, the expected value of the quantity on hand at the end of two days into the review period is given by $E[I_t] - 2\mu$ or $1.5\mu + C_2$. But the order quantity decision is made two days into the review period, and there is no outstanding order at this time. Thus the expected value of accurate quantity on record information is $1.5\mu + C_2$ or $q = 1106$. The accurate value of the expected order-up-to point information is then $r = q + d = 7.5\mu + C_2$ or 4250.

With a supply lead time of 4 days, demand for four days can be backordered so that accurate order-up-to point is reduced to $r = 7.5\mu + C_2 - 4\mu = 3.5\mu + C_2$ or $r = 2154$.

APPENDIX 2:

COMPUTER PROGRAMS

The computer programs used in this research are coded in GPSS/H Release 1, 1977. Each program is identified with the relevant section(s) in the text of this dissertation.

A2.1 Simulation Program for Steady State Detection:

This program, written in GPSS/H, is used in Sections 5.2 and 5.3 to make pilot simulation runs.

SIMULATION

*

* SAVEVALUES USED IN THE MODEL:

* X1 - QUANTITY ON RECORD ERROR	X2 - ORDER-UP-TO POINT ERROR
* X3 - REVIEW PERIOD DEMAND	X4 - QUANTITY ON HAND
* X5 - $\bar{d} = 1/N[\sum d_i]$	X6 - $\bar{r} = 1/N[\sum r_i]$
* X7 - $\bar{q} = 1/N[\sum q_i]$	X8 - $1/N[\sum \Delta d_i^2]$
* X9 - $1/N[\Delta r_i^2]$	X10 - $1/N[\sum \Delta q_i^2]$
* X11 - NET CONTRIBUTION	X12 - REVIEW PERIOD
* X13 - T_r	X14 - T_q
* X15 - ORDER-UP-TO POINT	X16 - QUANTITY ON RECORD
* X17 - CUMULATES Δd_i	X18 - CUMULATES Δr_i
* X19 - CUMULATES Δq_i	X20 - CUMULATES INVENTORY COST
* X21 - EPSILON (T)	X22 - INVENTORY COST
* X23 - DEMAND DURING PAST T	X24 - INTERVAL DEMAND

*

* SET RANDOM NUMBER SEEDS

*

RMULT 15011,81647,69179

*

* STANDARD NORMAL DISTRIBUTION FOR DAILY DEMAND

*

1 FUNCTION RN1,C25

0,-5/.00003,-4/.00135,-3/.00621,-2.5/.02275,-2
 .06681,-1.5/.11507,-1.2/.15886,-1/.21186,-.8/.27425,-.6
 .34458,-.4/.42074,-.2/.5,0/.57926,.2/.65542,.4
 .72575,.6/.78814,.8/.84134,1/.88493,1.2/.93319,1.5
 .97725,2/.99379,2.5/.99865,3/.99997,4/1,5

*

* STANDARD NORMAL DISTRIBUTION FOR THE ERROR

* IN QUANTITY ON RECORD INFORMATION

*

2 FUNCTION RN2,C25

0,-5/.00003,-4/.00135,-3/.00621,-2.5/.02275,-2
 .06681,-1.5/.11507,-1.2/.15886,-1/.21186,-.8/.27425,-.6
 .34458,-.4/.42074,-.2/.5,0/.57926,.2/.65542,.4
 .72575,.6/.78814,.8/.84134,1/.88493,1.2/.93319,1.5
 .97725,2/.99379,2.5/.99865,3/.99997,4/1,5

*

* STANDARD NORMAL DISTRIBUTION FOR ERROR

* IN ORDER-UP-TO POINT INFORMATION

*

3 FUNCTION RN3,C25

0, -5/.00003, -4/.00135, -3/.00621, -2.5/.02275, -2
.06681, -1.5/.11507, -1.2/.15886, -1/.21186, -.8/.27425, -.6
.34458, -.4/.42074, -.2/.5, 0/.57926, .2/.65542, .4
.72575, .6/.78814, .8/.84134, 1/.88493, 1.2/.93319, 1.5
.97725, 2/.99379, 2.5/.99865, 3/.99997, 4/1,5

*

* INITIALIZE STORAGE, MATRIX, AND FULLWORD SAVEVALUES

*

	STORAGE	S1,5/S2,4	STORAGE CAPACITY
1	MATRIX	X,2,4	SAVES ORDER QUANTITY AND DEMAND
	INITIAL	X1,100/X2,200/X4,320/X12,2/X13,4/X14,1	

* INITIALIZATION OF SAVEVALUES

*

* VARIABLE DEFINITIONS

1	FVARIABLE	524+153*FN1	CALCULATES DAILY DEMAND
2	FVARIABLE	X1*FN2*(X12+4-X14)	CALCULATES Δq_i
3	VARIABLE	((MX1(2,1)+MX1(2,2)+MX1(2,3)+MX1(2,4))*(X12+4-X14))/4	
*			CALCULATES q_i
4	FVARIABLE	1.65*X2+X2*FN3	CALCULATES Δr_i
5	VARIABLE	P7-((P7-1)/4)*4	CALCULATES COLUMN NO. OF MX1
6	VARIABLE	MX1(1,1)+MX1(1,2)+MX1(1,3)+MX1(1,4)	

* CALCULATES QUANTITY ON ORDER

7	VARIABLE	P7@X12	V7 - 1 : REVIEW PERIOD
8	FVARIABLE	(X17+X8/30)/(N\$DATA/30)	CALCULATES $1/N(\sum \Delta d_i^2)$
9	FVARIABLE	(X18+X9/30)/(N\$ORDER/30)	CALCULATES $1/N(\sum \Delta r_i^2)$
10	FVARIABLE	(X19+X10/30)/(N\$STOCK/30)	CALCULATES $1/N(\sum \Delta q_i^2)$
11	FVARIABLE	10000-2*(X20+X11/25)/(X5/250)	CALCULATES NET CONT

12	VARIABLE	10000*X11+X5	DATA ENCODING
13	VARIABLE	10000*X6+X7	DATA ENCODING
14	VARIABLE	10000*X12+100*X13+X14	DATA ENCODING
15	VARIABLE	MX1(2,1)+MX1(2,2)+MX1(2,3)+MX1(2,4)	LEAD TIME DEMAND
16	VARIABLE	(P7-6)@X12	CALCULATION OF EPSILON (T)
17	VARIABLE	(P7@X12+X12-X14)@X12	
QGEN	BVARIABLE	X12'E'1+V17'E'1	
CHECK	BVARIABLE	X12'E'1+V7'E'1	SCHEDULES DECISIONS
* MODEL SEGMENT 1 : BASIC OPERATIONS			
	GENERATE	1,,,1,7,F	DAY BEGINS
	ASSIGN	7,N1	P7 - DAY
DEMND	ASSIGN	1,V1	P1 - DEMAND
	TEST GE	P1,0,DEMND	NEGATIVE DEMAND?
	TEST G	P7,5,WAIT	IS DAY > 5?
	ASSIGN	4,5	P4 - 5
	SAVEVALUE	21+,V16*P1	CALCULATION OF EPSILON
	SAVEVALUE	3+,P1	STORE CURRENT DEMAND
WAIT	ENTER	1	ENTER STORAGE 1
	PRIORITY	0	LOWER PRIORITY
	TEST L	P7,6,STORE	INITIAL PERIOD?
	TEST E	P7,1,NEXT	FIRST DAY?
	SAVEVALUE	16,(X12-X14)*524	CORRECTION FACTOR
	TRANSFER	,DONE	SWITCH TO NEXT PART
NEXT	SAVEVALUE	16+,P1	INITIAL QUANTITY ON REC INFO
DONE	SAVEVALUE	4+,P1	DETERMINE INITIAL STOCK
	ENTER	2	ENTER STORAGE 2
	ADVANCE	5-P7	INTRODUCE DELAY

	ASSIGN	4, P7+X12-1	LENGTH OF NEXT DELAY
	LEAVE	2	LEAVE STORAGE 2
STORE	MSAVEVALUE	1, 2, V5, P1	SAVE DAILY DEMAND
	ASSIGN	2, X4-320-P1	MAGNITUDE OF DEVIATION
	TEST L	X4, P1, SHIPN	SUPPLY < DEMAND
	TEST G	P7, X12*0+5, SOUT	CALCULATE COST?
	SAVEVALUE	22+, 500*(P1-X4)	LOST SALES
SOUT	ASSIGN	2, 320	DEVIATION = 320
	SAVEVALUE	4, 0	NO MORE STOCK LEFT
	TRANSFER	, SKIP	GO TO SKIP
SHIPN	SAVEVALUE	4-, P1	REDUCE STOCK
SKIP	ASSIGN	3, X4	P3 - QUANTITY ON HAND
	TEST G	P7, 0*X12+5, HALT	CALCULATE COST?
	SAVEVALUE	22+, .00825*P2*P2	CUMULATE COST
	TEST E	(P7-X12-5)@X12, 0, HALT	END OF REVIEW PERIOD?
	HELPA	SHOWK, X22, X23	CALCULATE NET CONT
	SAVEVALUE	11+, X22	CUMULATE INTERVAL COST
	SAVEVALUE	22, 0	RESET REVIEW PERIOD COST
	SAVEVALUE	24+, X23	CUMULATE INTERVAL DEMAND
	TEST E	(P7-5)@(X12*25), 0, HALT	INTERVAL OVER?
	SAVEVALUE	20+, X11/25	CUMULATE TOTAL COST
	HELPA	SHOWI, X11, X24, X20, X5	CALCULATE INTERVAL CONT
	SAVEVALUE	11, 0	RESET INTERVAL STAT
	SAVEVALUE	24, 0	RESET INTERVAL STAT
	TEST L	P7, 625*X12+5, BYE	SIMULATION OVER?
HALT	ADVANCE	P4	LEAD TIME
	LEAVE	1	LEAVE STORAGE 1

* MODEL SEGMENT 2: DECISION MAKING

*

	TEST E	BV\$QGEN,1,RVIEW	GENERATE Q'?
	TEST L	N\$DECN,625,EXIT	NO MORE DECISIONS?
QERR	ASSIGN	5,V2	$P5 = \Delta q_i$
	TEST GE	P3+P5+V6,0,QERR	$Q' < 0?$
	SAVEVALUE	16,P3+P5+V6	$P5 = Q'$
	TEST GE	N\$DECN,0,RVIEW	COLLECT STATS?
STOCK	SAVEVALUE	7+,V3	CUMULATE q_i
	SAVEVALUE	10+,P5*P5	CUMULATE $(\Delta q_i)^2$
RVIEW	TEST E	1,BV\$CHECK,EXIT	REVIEW TIME?
	ASSIGN	5,0	EPSILON = 0
	TEST G	320*X12,X21,EXTRA	IS EPSILON > 0?
	ASSIGN	5,320-X21/X12	CALCULATE EPSILON
EXTRA	SAVEVALUE	21,0	RESET X21
	ASSIGN	2,X3-X4+P5	CALCULATE d_i
	TEST L	P2,0,OK	IS $d_i < 0?$
	ASSIGN	2,0	SET $d_i = 0$
OK	TEST GE	P7,N\$ORDER*X13+1,DECN	GENERATE R'?
	ASSIGN	3,P3+V6	$P3 = Q$
	ASSIGN	4,P2+P3	CALCULATE r_i
RERR	ASSIGN	5,V4	$P5 = \Delta r_i$
	TEST GE	P4+P5,0,RERR	$R' < 0?$
	SAVEVALUE	15,P4+P5	$X15 = R'$
	TEST GE	N\$DECN,0,DECN	COLLECT STAT?
ORDER	SAVEVALUE	6+,V15+X3	CUMULATE r_i
	SAVEVALUE	9+,P5*P5	CUMULATE $(\Delta r_i)^2$

DECN	ASSIGN	6,X15-X16+(X12-X14)*524	P6 = D'
	TEST L	P6,0,ARRVL	IS D' < 0?
	ASSIGN	6,0	SET D' = 0
ARRVL	SAVEVALUE	4+,P6	INCREASE STOCK
	TEST G	N\$DECN,0,RESET	COLLECT STAT?
DATA	SAVEVALUE	5+,X3	CUMULATE d_i
	ASSIGN	1,P2-P6	$P1 = \Delta d_i$
	SAVEVALUE	8+,P1*P1	CUMULATE $(\Delta d_i)^2$
	SAVEVALUE	23,X3	SAVE REVIEW PERIOD DEMAND
RESET	SAVEVALUE	3,0	RESET REVIEW PERIOD DEMAND
	TEST E	N\$DECN@30,0,EXIT	DATA TRANSFER?
	SAVEVALUE	17+,X8/30	STORE $(\Delta d_i)^2$
	SAVEVALUE	18+,X9/30	STORE $(\Delta r_i)^2$
	SAVEVALUE	19+,X10/30	STORE $(\Delta q_i)^2$
	SAVEVALUE	8,0	RESET D ERROR
	SAVEVALUE	9,0	RESET R ERROR
	SAVEVALUE	10,0	RESET Q ERROR
EXIT	MSAVEVALUE	1,1,V5,P6	STORE QTY. ON ORDER
	TERMINATE		THIS DAY OVER

*

* MODEL SEGMENT : FINAL STATS

*

BYE	SAVEVALUE	11,V11	CALCULATE CONTRIBUTION
	SAVEVALUE	5,X5/N\$DATA	$X5 = 1/N[\sum d_i]$
	SAVEVALUE	6,X6/N\$ORDER	$X6 = 1/N[\sum r_i]$
	SAVEVALUE	7,X7/N\$STOCK	$X7 = 1/N[\sum q_i]$
	SAVEVALUE	8,V8	$X8 = 1/N[\sum \Delta d_i^2]$

```

SAVEVALUE 9,V9          X9 = 1/N[ $\sum \Delta r_i^2$ ]
SAVEVALUE 10,V10       X10 = 1/N[ $\sum \Delta q_i^2$ ]
HELPA      SHOWA,V12,V13,V14,X8,X9,X10  CALCULATE STATS
TERMINATE 1            SIMULATION OVER
START      1,NP        START THIS RUN
END                THIS RUN OVER

```

A2.2 Simulation Program for the Lost Sales Case

This program, written in GPSS/H, is used to simulate the lost sales case described in Sections 5.4 through 5.6. This simulation program is based on antithetic variance reduction technique. The antithetic run is denoted by (A) in this program.

SIMULATE

* SAVEVALUES USED IN THE MODEL:

* X1 - QUANTITY ON RECORD ERROR	X2 - ORDER-UP-TO POINT ERROR
* X3 - REVIEW PERIOD DEMAND	X4 - QUANTITY ON HAND
* X5 - $d = 1/N[\sum d_i]$	X6 - $r = 1/N[\sum r_i]$
* X7 - $q = 1/N[\sum q_i]$	X8 - $1/N[\sum \Delta d_i^2]$
* X9 - $1/N[\Delta r_i^2]$	X10 - $1/N[\sum \Delta q_i^2]$
* X11 - NET CONTRIBUTION	X12 - REVIEW PERIOD
* X13 - T_r	X14 - T_q
* X15 - ORDER-UP-TO POINT	X16 - QUANTITY-ON-RECORD
* X17 - CUMULATES NET CONT	X18 - CUMULATES Δd_i^2
* X19 - CUMULATES Δr_i^2	X20 - CUMULATES Δq_i^2
* X21 - REVIEW PERIOD DEMAND (A)	X22 - QUANTITY ON HAND (A)
* X23 - ORDER-UP TO POINT (A)	X24 - QUANTITY-ON-RECORD (A)
* X25 - NET CONT (A)	X26 - CUMULATES NET CONT (A)
* X27 - $1/N[\sum \Delta d_i^2]$ (A)	X28 - CUMULATES Δd_i^2

* X29 = $1/N[\sum \Delta r_i^2]$ (A) X30 = CUMULATES Δr_i^2
 * X31 = $1/N[\sum \Delta q_i^2]$ (A) X32 = CUMULATES Δr_i^2 (A)
 * X33 = $d = 1/N[\sum d_i]$ (A) X34 = $r = 1/N[\sum x_i]$ (A)
 * X35 = $q = 1/N[\sum q_i]$ (A)
 * X36 = EPSILON (T) X37 = EPSILON (T) (A)
 * SET RANDOM NUMBER SEEDS

RMULT 15011,81647,69179

* STANDARD NORMAL DISTRIBUTION FOR DAILY DEMAND

*

1 FUNCTION RN1,C25

0, -5/.00003, -4/.00135, -3/.00621, -2.5/.02275, -2
 .06681, -1.5/.11507, -1.2/.15886, -1/.21186, -.8/.27425, -.6
 .34458, -.4/.42074, -.2/.5, 0/.57926, .2/.65542, .4
 .72575, .6/.78814, .8/.84134, 1/.88493, 1.2/.93319, 1.5
 .97725, 2/.99379, 2.5/.99865, 3/.99997, 4/1,5

*

* STANDARD NORMAL DISTRIBUTION FOR THE ERROR

* IN QUANTITY ON RECORD INFORMATION

*

2 FUNCTION RN2,C25

0, -5/.00003, -4/.00135, -3/.00621, -2.5/.02275, -2
 .06681, -1.5/.11507, -1.2/.15886, -1/.21186, -.8/.27425, -.6
 .34458, -.4/.42074, -.2/.5, 0/.57926, .2/.65542, .4
 .72575, .6/.78814, .8/.84134, 1/.88493, 1.2/.93319, 1.5
 .97725, 2/.99379, 2.5/.99865, 3/.99997, 4/1,5

*

* STANDARD NORMAL DISTRIBUTION FOR ERROR

* IN ORDER-UP-TO POINT INFORMATION

*

3 FUNCTION RN3,C25

0, -5/.00003, -4/.00135, -3/.00621, -2.5/.02275, -2

.06681, -1.5/.11507, -1.2/.15886, -1/.21186, -.8/.27425, -.6

.34458, -.4/.42074, -.2/.5, 0/.57926, .2/.65542, .4

.72575, .6/.78814, .8/.84134, 1/.88493, 1.2/.93319, 1.5

.97725, 2/.99379, 2.5/.99865, 3/.99997, 4/1,5

*

* INITIALIZE STORAGE, MATRIX AND FULLWORD SAVEVALUES

*

	STORAGE	S1,5/S2,4	STORAGE CAPACITY
1	MATRIX	X,2,4	SAVES ORDER QTY. AND DEMAND
2	MATRIX	X,2,4	SAVES ORDER QTY., DEMAND (A)
	INITIAL	X1,105/X2,420/X4,320/X12,6/X13,6/X14,5/X22,320	

* INITIALIZATION OF SAVEVALUES

* VARIABLE DEFINITIONS:

*

1	FVARIABLE	524+153*FN1	CALCULATES DAILY DEMAND
2	FVARIABLE	X1*FN2*(X12+4-X14)	CALCULATES Δq_i
3	VARIABLE	((MX1(2,1)+MX1(2,2)+MX1(2,3)+MX1(2,4))*(X12+4-X14))/4	
			CALCULATES q_i
4	FVARIABLE	1.65*X2+X2*FN3	CALCULATES Δr_i
5	VARIABLE	P7-((P7-1)/4)*4	CALCULATES COLUMN NO. OF MX1
6	VARIABLE	MX1(1,1)+MX1(1,2)+MX1(1,3)+MX1(1,4)	

* CALCULATES QUANTITY ON ORDER

7	VARIABLE	P7@X12	V7 - 1 : REVIEW TIME
8	FVARIABLE	(X18+X8/30)/(N\$DATA/30)	CALCULATES $1/N[\sum \Delta d_i^2]$
9	FVARIABLE	(X19+X9/30)/(N\$ORDER/30)	CALCULATES $1/N[\sum \Delta r_i^2]$
10	FVARIABLE	(X20+X10/30)/(N\$STOCK/30)	CALCULATES $1/N[\sum \Delta q_i^2]$
11	FVARIABLE	10000-2*(X17+X11/30)/(X5/300)	NET CONT.
12	VARIABLE	X11*10000+X5	DATA ENCODING
13	VARIABLE	10000*X6+X7	DATA ENCODING
14	VARIABLE	X12*10000+X13*100+X14	DATA ENCODING
15	FVARIABLE	1048-P1	CALCULATES DAILY DEMAND (A)
16	FVARIABLE	0-P3	CALCULATES Δq_i (A)
17	VARIABLE	MX2(1,1)+MX2(1,2)+MX2(1,3)+MX2(1,4)	
*			QUANTITY ON ORDER (A)
18	VARIABLE	((MX2(2,1)+MX2(2,2)+MX2(2,3)+MX2(2,4))*(X12+4-X14))/4	
*			CALCULATES q_i (A)
19	FVARIABLE	3.3*X2-P4	CALCULATES Δr_i (A)
20	FVARIABLE	10000-2*(X26+X25/30)/(X33/300)	NET CONT. (A)
21	FVARIABLE	(X28+X27/30)/N\$DATA/30)	CALCULATES $1/N[\sum \Delta d_i^2]$ (A)
22	FVARIABLE	(X30+X29/30)/(N\$ORDER/30)	CALCULATES $1/N[\sum \Delta r_i^2]$ (A)
23	FVARIABLE	(X32+X31/30)/(N\$STOCK/30)	CALCULATES $1/N[\sum \Delta q_i^2]$ (A)
24	VARIABLE	X25*10000+X33	DATA ENCODING (A)
25	VARIABLE	10000*X34+X35	DATA ENCODING (A)
26	VARIABLE	(X12-X14)*524	CORRECTION FACTOR
27	VARIABLE	MX1(2,1)+MX1(2,2)+MX1(2,3)+MX1(2,4)	LEAD TIME DEMAND
28	VARIABLE	MX2(2,1)+MX2(2,2)+MX2(2,3)+MX2(2,4)	LEAD TIME DEM (A)
29	VARIABLE	(P7-6)@X12	USED IN EPSILON (T)
30	VARIABLE	(P7@X12-X14+X12)@X12	USED IN QGEN BVARIABLE
QGEN	BVARIABLE	X12'E'1+V30'E'1	TIME TO UPDATE q'_i ?

CHECK	BVARIABLE	X12'E'1+V7'E'1	SCHEDULES DECISIONS
DMD	BVARIABLE	P1'GE'0*P2'GE'0	CHECKS DEMAND
QDEV	BVARIABLE	X16'GE'0*X24'GE'0	CHECKS QUANTITY ON RECORD
RDEV	BVARIABLE	X15'GE'0*X23'GE'0	CHECKS ORDER UP TO POINT
*			
* MODEL SEGMENT 1: BASIC OPERATIONS			
*			
	GENERATE	1, , , , 1, 7, F	DAY BEGINS
	ASSIGN	7, N1	P7 - DAY
DEMND	ASSIGN	1, V1	CALCULATE DEMAND
	ASSIGN	2, V15	CALCULATE DEMAND (A)
	TEST E	1, BV\$DMD, DEMND	NEGATIVE DEMAND ?
	TEST G	P7, 5, WAIT	IS DAY > 5 ?
	ASSIGN	4, 5	P4 - 5
	SAVEVALUE	36+, V29*P1	CALCULATION OF EPSILON (T)
	SAVEVALUE	37+, V29*P2	CALCULATES EPSILON (T) (A)
	SAVEVALUE	3+, P1	STORE CURRENT DEMAND
	SAVEVALUE	21+, P2	STORE CURRENT DEMAND (A)
WAIT	ENTER	1	ENTER STORAGE 1
	PRIORITY	0	LOWER PRIORITY
	TEST L	P7, 6, STORE	INITIAL PERIOD?
	TEST E	P7, 1, NEXT	DAY ONE?
	SAVEVALUE	16, V26	CORRECTION FACTOR
	SAVEVALUE	24, V26	CORRECTION FACTOR (A)
	TRANSFER	, DONE	GO TO DONE
NEXT	SAVEVALUE	16+, P1	INITIAL QTY ON RECORD
	SAVEVALUE	24+, P2	INITIAL QTY ON RECORD (A)

DONE	SAVEVALUE	4+, P1	DETERMINE INITIAL INVENTORY
	SAVEVALUE	22+, P2	DETERMINE INITIAL INV. (A)
	ENTER	2	ENTER STORAGE 2
	ADVANCE	5-P7	INTRODUCE DELAY
	ASSIGN	4, P7+X12-1	LENGTH OF NEXT DELAY
	LEAVE	2	LEAVE STORAGE 2
STORE	MSAVEVALUE	1, 2, V5, P1	SAVE DAILY DEMAND
	MSAVEVALUE	2, 2, V5, P2	SAVE DAILY DEMAND (A)
	ASSIGN	3, X4-320-P1	P3-MAGNITUDE OF DEVIATION
	ASSIGN	5, X22-320-P2	P5-MAGNITUDE OF DEV (A)
	TEST L	X4, P1, SHIPN	SUPPLY < DEMAND?
	TEST G	P7, X12*50+5, LOSTS	STEADY STATE?
	SAVEVALUE	11+, 500*(P1-X4)	RECORD LOST SALES
LOSTS	ASSIGN	3, 320	RECORD DEVIATION
	SAVEVALUE	4, 0	NO MORE STOCK LEFT
	TRANSFER	, SKIP	GO TO SKIP
SHIPN	SAVEVALUE	4-, P1	REDUCE STOCK
SKIP	ASSIGN	1, X4	P1 - QUANTITY ON HAND
	TEST L	X22, P2, SHIPA	SUPPLY < DEMAND? (A)
	TEST G	P7, X12*50+5, SLOST	STEADY STATE (A)?
	SAVEVALUE	25+, 500*(P2-X22)	RECORD LOST SALES (A)
SLOST	ASSIGN	5, 320	RECORD DEVIATION (A)
	SAVEVALUE	22, 0	NO MORE STOCK LEFT (A)
	TRANSFER	, SKIPA	GO TO SKIPA
SHIPA	SAVEVALUE	22-, P2	REDUCE STOCK (A)
SKIPA	ASSIGN	2, X22	P2 - QUANTITY ON HAND (A)
	TEST G	P7, X12*50+5, HALT	CALCULATE CONT. ?

	SAVEVALUE	11+, .00825*P3*P3	SUM INV COST
	SAVEVALUE	25+, .00825*P5*P5	SUM INV COST (A)
	TEST L	P7, 350*X12+5, BYE	SIMULATION OVER?
HALT	ADVANCE	P4	LEAD TIME+REVIEW TIME
	ASSIGN	4, 0	RESET P4
	LEAVE	1	LEAVE STORAGE

*

* MODEL SEGMENT 2 : DECISION MAKING

*

	TEST E	BV\$QGEN, 1, RVIEW	GENERATE q'_i ?
	TEST L	N\$DECN, 350, CALL	NO MORE DECN?
QURR	ASSIGN	3, V2	$P3 - \Delta q_i$
	ASSIGN	5, V16	$P5 - \Delta q_i (A)$
	SAVEVALUE	16, P3+P1+V6	$X16 - q'_i$
	SAVEVALUE	24, P5+P2+V17	$X24 - q'_i (A)$
	TEST E	1, BV\$QDEV, QERR	$q'_i > 0?$
	TEST GE	N\$DECN, 50, RVIEW	COLLECT STATS?
STOCK	SAVEVALUE	7+, V3	SUM q_i
	SAVEVALUE	35+, V18	SUM $q_i (A)$
	SAVEVALUE	10+, P3*P3	SUM Δq_i^2
	SAVEVALUE	31+, P5*P5	SUM $\Delta q_i^2 (A)$
RVIEW	TEST E	1, BV\$CHECK, CALL	REVIEW TIME ?
	ASSIGN	3, 0	$P3 - 0$
	ASSIGN	5, 0	$P5 - 0$
	TEST G	320*X12, X36, EXTRA	$\epsilon_i > 0 ?$
	ASSIGN	3, 320-X36/X12	$P3 - \epsilon_i$
EXTRA	SAVEVALUE	36, 0	RESET X36

	TEST G	320*X12,X37,EXTRB	$\epsilon_i > 0 ?$
	ASSIGN	5,320-X37/X12	$P5 = \epsilon_i$
EXTRB	SAVEVALUE	37,0	RESET X37
	ASSIGN	3,X3-X4+P3	CALCULATE d_i
	ASSIGN	5,X21-X22+P5	CALCULATE d_i (A)
	TEST L	P3,0,OKA	IS $d_i < 0 ?$
	ASSIGN	3,0	SET $d_i = 0$
OKA	TEST L	P5,0,OK	IS $d_i < 0 ?$ (A)
	ASSIGN	5,0	SET $d_i = 0$ (A)
OK	TEST GE	P7,N\$ORDER*X13+1,DECN	GENERATE $r_i ?$
	ASSIGN	1,P1+V6	$P1 = q_i$
	ASSIGN	2,P2+V17	$P2 = q_i$ (A)
	ASSIGN	1,P1+P3	$P1 = r_i$
	ASSIGN	2,P2+P5	$P2 = r_i$ (A)
RERR	ASSIGN	4,V4	$P4 = \Delta r_i$
	ASSIGN	6,V19	$P6 = \Delta r_i$ (A)
	SAVEVALUE	15,P4+P1	$X15 = r'_i$
	SAVEVALUE	23,P2+P6	$X23 = r'_i$ (A)
	TEST E	1,BV\$RDEV,RERR	$r'_i \geq 0?$
	TEST GE	N\$DECN,50,DECN	COLLECT STAT?
ORDER	SAVEVALUE	6+,V27+X3	SUM r_i
	SAVEVALUE	34+,V28+X21	SUM r_i (A)
	SAVEVALUE	9+,P4*P4	SUM Δr_i^2
	SAVEVALUE	29+,P6*P6	SUM Δr_i^2 (A)
DECN	ASSIGN	4,X15-X16+V26	$P4 = d'_i$
	TEST L	P4,0,ARRVA	$d'_i < 0 ?$
	ASSIGN	4,0	$d'_i = 0$

ARRVA	ASSIGN	6,X23-X24+V26	$P6 = d'_i$ (A)
	TEST L	P6,0,ARRVL	$d'_i < 0 ?$ (A)
	ASSIGN	6,0	$d'_i = 0$ (A)
ARRVL	SAVEVALUE	4+P4	INCREASE STOCK
	SAVEVALUE	22+,P6	INCREASE STOCK (A)
	TEST G	N&DECN,50,RESET	COLLECT STAT?
DATA	SAVEVALUE	5+,X3	SUM d_i
	SAVEVALUE	33+,X21	SUM d_i (A)
	ASSIGN	1,P3-P4	$P1 = \Delta d_i$
	ASSIGN	2,P5-P6	$P5 = \Delta d_i$ (A)
	SAVEVALUE	8+,P1*P1	SUM Δd_i^2
	SAVEVALUE	27+,P2*P2	SUM Δd_i^2 (A)
RESET	SAVEVALUE	3,0	INITIALIZE X3
	SAVEVALUE	21,0	INITIALIZE X21 (A)
	TEST E	N\$DECN@30,0,CALL	DATA TRANSFER?
	SAVEVALUE	17+,X11/30	STORE RESULT
	SAVEVALUE	18+,X8/30	STORE RESULT
	SAVEVALUE	19+,X9/30	STORE RESULT
	SAVEVALUE	20+,X10/30	STORE RESULT
	SAVEVALUE	8,0	RESET DATA
	SAVEVALUE	9,0	RESET DATA
	SAVEVALUE	10,0	RESET DATA
	SAVEVALUE	11,0	RESET DATA
	SAVEVALUE	26+,X25/30	STORE RESULT (A)
	SAVEVALUE	28+,X27/30	STORE RESULT (A)
	SAVEVALUE	30+,X29/30	STORE RESULT (A)
	SAVEVALUE	32+,X31/30	STORE RESULT (A)

SAVEVALUE	25,0	RESET DATA (A)
SAVEVALUE	27,0	RESET DATA (A)
SAVEVALUE	29,0	RESET DATA (A)
SAVEVALUE	31,0	RESET DATA (A)
CALL	MSAVEVALUE 1,1,V5,P4	SAVE ORDER QUANTITY
	MSAVEVALUE 2,1,V5,P6	SAVE ORDER QUANTITY
	TERMINATE	FINISH
* MODEL SEGMENT 3 : FINAL STATS		
*		
BYE	SAVEVALUE 11,V11	NET CONT.
	SAVEVALUE 25,V20	NET CONT. (A)
	SAVEVALUE 5,X5/N\$DATA	$1/N[\sum d_i]$
	SAVEVALUE 33,X33/N\$DATA	$1/N[\sum d_i] (A)$
	SAVEVALUE 6,X6/N\$ORDER	$1/N[\sum r_i]$
	SAVEVALUE 34,X34/N\$ORDER	$1/N[\sum r_i] (A)$
	SAVEVALUE 7,X7/N\$STOCK	$1/N[\sum q_i]$
	SAVEVALUE 35,X35/N\$STOCK	$1/N[\sum r_i] (A)$
	SAVEVALUE 8,V8	$X8 - 1/N[\sum \Delta d_i^2]$
	SAVEVALUE 27,V21	$X27 - 1/N[\sum \Delta d_i^2] (A)$
	SAVEVALUE 9,V9	$X9 - 1/N[\sum \Delta r_i^2] (R)$
	SAVEVALUE 29,V22	$X29 - 1/N[\sum \Delta r_i^2] (A)$
	SAVEVALUE 10,V10	$X10 - 1/N[\sum \Delta q_i^2] (Q)$
	SAVEVALUE 31,V23	$X31 - 1/N[\sum \Delta q_i^2] (A)$
HELPA	SHOWA,V12,V13,V14,X8,X9,X10 TO SUBROUTINE	
HELPA	SHOWA,V24,V25,V14,X27,X29,X31 TO SUBROUTINE	
TERMINATE	1	SIMULATION OVER
START	1,NP	START THIS RUN

A2.3 Fortran Subroutines

The Fortran subroutines used in the two GPSS/H programs listed above, SHOWA, SHOWI, and SHOWK, are documented in this section.

1. SUBROUTINE SHOWA

```

SUBROUTINE SHOWA(A)

C   A(1)=10000*X11+X5
C   A(2)=10000*X6+X7
C   A(3)=10000*X12*100*X13+X14
C   A(4)=X8, A(5)=X9, A(6)=X10.

INTEGER*4 A(6)

DIMENSION D(10), L(7)

K=0

DO 1 J=1,3

K=K+1

L(K)=A(J)/10000

K=K+1

1 L(K)=A(J)-10000*L(K-1)

L(K)=L(K)/100

K=K+1

L(K)=A(J)-10000*L(K-2)-100*L(K-1)

L(K)=L(K-2)-L(K)

DO 2 N=1,7

2 D(N)=L(N)

DO 3 N=8,10

3 D(N)=A(N-4)

D(1)=D(1)/100.0

DO 4 M=2,4

```

```

4 D(M)=100-100*SQRT(D(M+6))/D(M)
   DO 5 M=5,7
5 D(M)=100/D(M)
   WRITE(6,6) (D(M), M=1,7)
C   D(1)=NET CONT; D(2)=Da; D(3)=Ra; D(4)=Qa;
C   D(5)=Dc; D(6)=Rc; D(7)=Qc.
6 FORMAT(7F8.2)

RETURN

END

```

2. SUBROUTINE SHOWI

```

SUBROUTINE SHOWI(A)
C   A(1)=X11, A(2)=X24, A(3)=X20, A(4)=X5.
   INTEGER*4 A(4)
   DIMENSION B(6)
   DO 1 I=1,4
1 B(I)=A(I)
   B(5)=100.0-B(1)/(5.0*B(2))
   B(6)=100.0-5.0*B(3)/B(4)
   WRITE(6,2) (B(5), B(6))
2 FORMAT(2F8.2)

RETURN

END

```

3. SUBROUTINE SHOWK

```

SUBROUTINE SHOWK(A)
C   A(1)=INV COST; A(2)=DEMAND.
   INTEGER*4 A(2)
   D=A(1)

```

```
C=A(2)
B=10000.0-2.0*(10.0*D/C)
B=B/100.0
WRITE(5,1) B
1 FORMAT(F8.2)
RETURN
END
```

APPENDIX 3:

ESTIMATION OF PRODUCTION FUNCTIONS

A3.1 Properties of the Translog Function:

The translog function in three inputs can be written as:

$$(A3.1) \ln Y = a_0 + \sum_m a_m \ln x_m + \sum_m \sum_n a_{mn} \ln x_m \ln x_n$$

where $m, n = 1, 2, 3$; and $m \leq n$. The first and second order partial derivatives of (A3.1) are given below:

$$(A3.2) \frac{\delta Y}{\delta x_m} = \frac{Y}{x_m} [a_m + \sum_n a_{mn} X_n]$$

$$(A3.3) \frac{\delta^2 Y}{\delta x_m^2} = \frac{Y}{x_m^2} [A_m + \sum_n A_{mn} X_n + (\sum_n a_{mn} X_n)^2]$$

$$\text{where } X_n = 2 \ln x_n \text{ if } m = n, \\ = \ln x_n \text{ otherwise;}$$

$$A_m = a_m^2 - a_m + 2 a_{mm};$$

$$A_{mn} = (2 a_m - 1) a_{mn};$$

$$\text{and } m, n = 1, 2, 3.$$

The elasticity of substitution σ_{mn} between x_m and x_n and the cross derivatives of (A3.1) are given by (A3.4) and (A3.5) respectively:

$$(A3.4) \sigma_{mn} = \frac{x'_m + x'_n}{2a_{mn} + x'_m + x'_n - 2a_{mm}(x'_n/x'_m) - 2a_{nn}(x'_m/x'_n)};$$

$$(A3.5) \frac{\delta^2 Y}{\delta x_m \delta x_n} = \frac{Y}{x_m x_n} [a_{mn} + x'_m x'_n]$$

$$\text{where } x'_p = a_p + \sum_q a_{pq} X_q;$$

$$\text{and } p = m, n; q = 1, 2, 3.$$

Finally, the condition for increasing returns to scale is given as:

$$(A3.6) \sum_m a_m + \sum_m \ln x_m [\sum_n a_{mn} + a_{mm}] > 1; \text{ where } m, n = 1, 2, 3.$$

A3.2 Interval Band of the Total Product Curves

The total product curves presented in Section 5.4 are based on point estimates of the dependent variables. It may be recalled that each data point in a curve is obtained from a number of independent replications of the simulation. Each data point then represents the average of these replications. In this section the interval estimates of the total product curves are discussed.

The variance of each data point is kept low using the steps described in Section 5.3. As a result, the estimated interval of each data point is fairly narrow. An example of an interval band of a total product curve is given in Table A3.1, and is illustrated in Figure A3.1. The interval estimates given in Table A3.1 are based on 90% limits of the standard normal distribution.

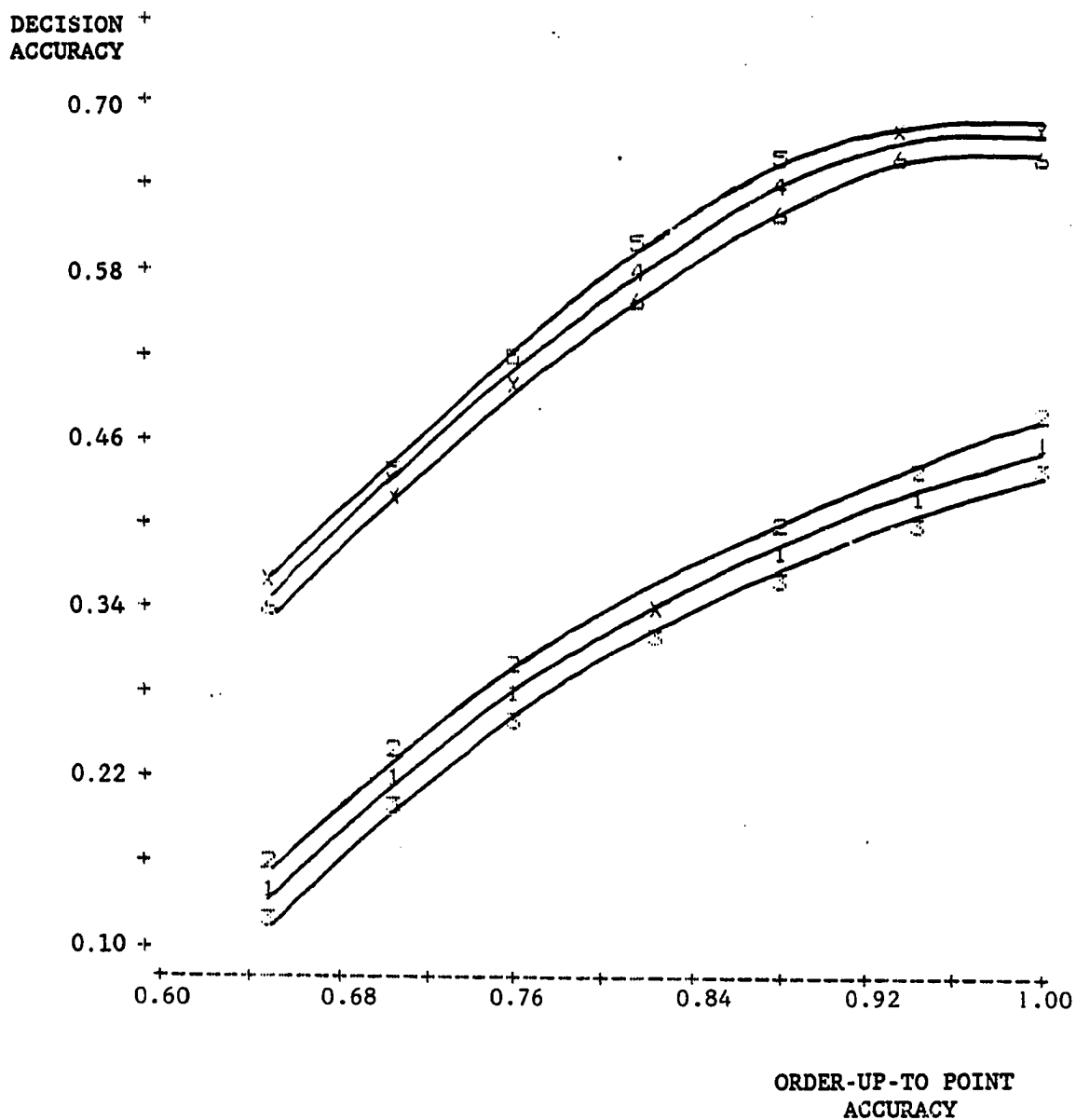
TABLE A3.1: TOTAL PRODUCT OF ORDER-UP-TO POINT ACCURACY (R_a)
FOR (1) $Q_a=0.55$, $R_c=0.40$, $Q_c=0.25$ AND (2) $Q_a=0.70$, $R_c=0.30$, $Q_c=0.20$

INPUT (R_a)	CONFIDENCE INTERVAL FOR INPUT COMBINATION #1			CONFIDENCE INTERVAL FOR INPUT COMBINATION #2		
	Upper	Average	Lower	Upper	Average	Lower
0.64	0.1534	0.1326	0.1118	0.3665	0.3525	0.3385
0.70	0.2322	0.2178	0.2034	0.4303	0.4209	0.4115
0.76	0.2939	0.2777	0.2615	0.5115	0.5011	0.4907
0.82	0.3449	0.3329	0.3209	0.5953	0.5796	0.5639
0.88	0.3938	0.3799	0.3660	0.6538	0.6411	0.6284
0.94	0.4365	0.4193	0.4021	0.6859	0.6719	0.6579
1.00	0.4711	0.4536	0.4361	0.6867	0.6743	0.6619

FIGURE A3.1: INTERVAL BAND FOR THE TOTAL PRODUCT CURVE OF
ORDER-UP-TO POINT ACCURACY

TOP BAND: Quantity on record accuracy = 0.70,
Quantity on record coverage = 0.20,
Order-up-to point coverage = 0.30

BOTTOM BAND: Quantity on record accuracy = 0.55,
Quantity on record coverage = 0.25,
Order-up-to point coverage = 0.40



A3.3 Estimation of Production Functions: Lost Sales Case

1. Gross Payoff Function: The OLS regressions of the estimated CES, VES, and Translog functions are given below:

(i) CES FUNCTION:

ANALYSIS OF VARIANCE OF 1. $\ln Y$ N = 100 OUT OF 100

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	3	4.9956	1.6652	33.827	.0000
ERROR	96	4.7258	.49227 -1		
TOTAL	99	9.7214			

MULT R = .71685 R-SQR = .51388 R-ADJ = .49869 SE = .22187

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		.33756	.69073 -1	4.8870	.0000
$\ln D_a$.47266	.20596	.39192 -1	5.2552	.0000
$\ln D_c$.70640	.36669	.37500 -1	9.7784	.0000
$[\ln D_a - \ln D_c]^2$.26511	.40157 -1	.14906 -1	2.6940	.0083

(ii) VES FUNCTION

ANALYSIS OF VARIANCE OF 1. $\ln (Y/D_a)$ N = 100 OUT OF 100

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	3	36.457	12.152	240.51	.0000
ERROR	96	4.8505	.50526 -1		
TOTAL	99	41.307			

MULT R = .93945 R-SQR = .88257 R-ADJ = .87890 SE = .22478

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		.31276	.71737 -1	4.3598	.0000
$\ln D_a$	-.56740	-.43231	.64032 -1	-6.7514	.0000
(D_c/D_a)	.21384	.34059 -1	.15880 -1	2.1448	.0345
$\ln (D_c/D_a)$.61691	.29140	.37943 -1	7.6800	.0000

(iii) TRANSLOG FUNCTION :

ANALYSIS OF VARIANCE OF $\ln Y$ N = 100 OUT OF 100					
SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	5	7.8441	1.5688	78.554	.0000
ERROR	94	1.8773	.19971	-1	
TOTAL	99	9.7214			

MULT R = .89827 R-SQR = .80689 R-ADJ = .79662 SE = .14132

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF	
CONSTANT		-.13178	.72677	-1	-1.8132	.0730
$\ln D_a$	-.05402	-.60670	.11568	-1	-.52448	.6012
$\ln D_c$	-.59376	-.62879	.87889	-1	-7.1544	.0000
$[\ln D_a]^2$.05402	.18375	.35033	-1	-.52450	.6012
$[\ln D_c]^2$	-.71993	-.31301	.31124	-1	-10.057	.0000
$[\ln D_a \ln D_c]$	-.59957	-.33610	.46272	-1	-7.2634	.0000

It is clear from above that the CES function provides a poor fit compared to the other two functions. The translog function provides good fit but has a multicollinearity problem. Although the estimated translog function is highly significant, some of its coefficients are not. The multicollinearity of the translog model is further evidenced by the fact that the highest R^2 delete is 0.806 which is very close to the original R^2 . The VES function, however, has no multicollinearity problem and has highest R^2 , and is therefore chosen as the final model. The marginal products of the VES function are:

$$\frac{\delta Y}{\delta D_a} = \alpha \theta \rho \frac{Y}{D_a + (\rho-1)D_c} = 0.277 \frac{Y}{D_a + 0.123 D_c} > 0;$$

$$\frac{\delta Y}{\delta D_c} = \alpha(1-\theta\rho) \frac{Y}{D_c} + \frac{\alpha\theta\rho(\rho-1) Y}{D_a+(\rho-1) D_c} = 0.291 \frac{Y}{D_c} + \frac{0.034 Y}{D_a+0.123 D_c} > 0.$$

Note that the marginal products diminish with increasing input levels.

2. Decision Generation Function:ANALYSIS OF VARIANCE OF 1. $\ln D_a$ N = 125 OUT OF 125

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	9	15.242	1.6936	1389.6	.0000
ERROR	115	.14016	.12187 -2		
TOTAL	124	15.382			

MULT R = .99543 R-SQR = .99089 R-ADJ = .99018 SE = .34910 -1

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		-.14122	.15559 -1	-9.0764	.0000
$\ln R_a$.61650	.69771	.83092 -1	8.3968	.0000
$\ln Q_a$	-.15636	-.13152	.77470 -1	-1.6977	.0923
$\ln Q_c$	-.57895	-.16657	.21876 -1	-7.6145	.0000
$[\ln R_a]^2$	-.83838	-2.5553	.15492	-16.494	.0000
$[\ln Q_a]^2$	-.61942	-1.1970	.14147	-8.4612	.0000
$[\ln Q_c]^2$	-.65378	-.10792	.11648 -1	-9.2653	.0000
$[\ln R_a \ln Q_a]$.13515	.18042	.12334	1.4628	.1463
$[\ln R_a \ln Q_c]$	-.02749	-.10453 -1	.35439 -1	-.29495	.7686
$[\ln Q_a \ln Q_c]$	-.87284	-.64286	.33517 -1	-19.180	.0000

The translog function estimated above exhibits excellent fit, although it has a multicollinearity problem as evidenced by the low level of significance of two of the estimated parameters. For reasons stated in Section 5.5, however, this function can be used for preliminary analysis performed in this research.

A3.4 Estimation of Production functions: Backordering Case

1. Gross Payoff Function: Of the three estimated functions, the CES model provides the lowest fit, and is rejected. The translog function provides the next best fit, but exhibits multicollinearity. The VES

function exhibits the best R^2 and adjusted R^2 values, and is chosen as the final model. The OLS regressions of the estimated CES, VES, and Translog functions are:

(i) CES FUNCTION:

ANALYSIS OF VARIANCE OF 1. $\ln Y$ N = 100 OUT OF 100

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	3	1.5148	.50492	47.561	.0000
ERROR	96	1.0192	.10616 -1		
TOTAL	99	2.5339			

MULT R = .77317 R-SQR = .59779 R-ADJ = .58522 SE = .10303

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		.18490	.31464 -1	5.8765	.0000
$\ln D_a$.54775	.14768	.23022 -1	6.4148	.0000
$\ln D_c$.70406	.17315	.17825 -1	9.7140	.0000
$[\ln D_a - \ln D_c]^2$.30580	.30583 -1	.97183 -2	3.1470	.0022

(ii) VES FUNCTION

ANALYSIS OF VARIANCE OF 1. $\ln (Y/D_a)$ N = 100 OUT OF 100

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	3	18.590	6.1967	569.67	.0000
ERROR	96	1.0443	.10878 -1		
TOTAL	99	19.634			

MULT R = .97304 R-SQR = .94681 R-ADJ = .94515 SE = .10430

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		.14234	.38584 -1	3.6892	.0004
$\ln D_a$	-.92236	-.67473	.28843 -1	-23.393	.0000
(D_c/D_a)	.26680	.45718 -1	.16855 -1	2.7124	.0079
$\ln (D_c/D_a)$.43738	.10223	.21453 -1	4.7655	.0000

(iii) TRANSLOG FUNCTION :

ANALYSIS OF VARIANCE OF 1. ln Y N = 100 OUT OF 100					
SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	5	2.1929	.43859	120.91	.0000
ERROR	94	.34097	.36273 -2		
TOTAL	99	2.5339			

MULT R = .93029 R-SQR = .86544 R-ADJ = .85828 SE = .60227 -1

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		-.99036 -1	.31037 -1	-3.1909	.0019
ln D _a	-.24442	-.14334	.58653 -1	-2.4439	.0164
ln D _c	-.64621	-.29669	.36139 -1	-8.2097	.0000
[ln D _a] ²	-.11870	-.28840 -1	.24890 -1	-1.1591	.2494
[ln D _c] ²	-.69697	-.12182	.12928 -1	-9.4231	.0000
[ln D _a ln D _c]	-.78165	-.22727	.18705 -1	-12.150	.0000

2. Decision Generation Function: The estimation of the decision generation function for this case is very similar to that of the lost sales case. An OLS regression model is used to estimate the translog function. The translog model given below is highly significant and provides excellent fit. However, the translog model, as before, exhibit some degree of multicollinearity as evidenced by the low level of significance of one of the estimated parameters. The estimated translog model, however, is used for preliminary analysis only, and so no steps are taken to alter the estimated function. Finally, the Cobb-Douglas alternative is tested by examining the null hypothesis that all quadratic terms in the translog model are simultaneously equal to zero. This hypothesis is rejected at 0.01 significance level indicating that the Cobb-Douglas alternative is not appropriate in this case.

ANALYSIS OF VARIANCE OF 1. $\ln D_a$ N = 125 OUT OF 125

SOURCE	DF	SUM SQRS	MEAN SQR	F-STAT	SIGNIF
REGRESSION	9	18.775	2.0861	560.18	.0000
ERROR	115	.42825	.37239 -2		
TOTAL	124	19.203			

MULT R = .98879 R-SQR = .97770 R-ADJ = .97595 SE = .61024 -1

VARIABLE	PARTIAL	COEFF	STD ERROR	T-STAT	SIGNIF
CONSTANT		-.18213	.25223 -1	-7.2206	.0000
$\ln R_a$.70264	.41065	.38779 -1	10.589	.0000
$\ln Q_a$	-.19284	-.16110	.76442 -1	-2.1075	.0372
$\ln Q_c$	-.60827	-.30800	.37478 -1	-8.2181	.0000
$[\ln R_a]^2$.10698	.22199 -1	.19239 -1	1.1539	.2509
$[\ln Q_a]^2$	-.30441	-.28195	.82271 -1	-3.4271	.0008
$[\ln Q_c]^2$	-.71836	-.22548	.20362 -1	-11.074	.0000
$[\ln R_a \ln Q_a]$	-.24840	-.91820 -1	.33390 -1	-2.7500	.0069
$[\ln R_a \ln Q_c]$	-.25982	.47623 -1	.16505 -1	-2.8853	.0047
$[\ln Q_a \ln Q_c]$	-.83521	-.55419	.34027 -1	-16.287	.0000

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